
Homework 9

Exercise 1 *Compute the curve integrals in the following situations:*

(i) $f : \mathbb{R}^2 \ni (x, y) \mapsto (x^2 - xy, y^2 - 2xy) \in \mathbb{R}^2$ *and the curve defined by the parabola* $y = x^2$ *from* $(-2, 4)$ *to* $(1, 1)$.

(ii) $f : \mathbb{R}^3 \ni (x, y, z) \mapsto (x, z, xz - y) \in \mathbb{R}^3$ *and the curve defined by the segment between* $(0, 0, 0)$ *and* $(1, 2, 4)$,

(iii) $f : \mathbb{R}^2 \setminus \{(0, 0)\} \ni (x, y) \mapsto \big(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\big)$ *and the curve defined by the circle centered at* $(0, 0)$ *and of radius* 2, *taken in counterclockwise direction.*

Exercise 2 *a) Consider the vector field* $f : \mathbb{R}^2 \ni (x, y) \mapsto (2xy, x^2 + y^2) \in \mathbb{R}^2$. *Compute the curve integral along the following curves: (i) The segment between* $(0, 0)$ *and* $(1, 1)$, *(ii) The parabola of equation* $y = x^2$ *from the point* $(0, 0)$ *to the point* $(1, 1)$. *What do you observe ?*

Exercise 3 *Compute the curve integral*

$$\int_C (2x - y)\mathrm{d}x + (x + y)\mathrm{d}y$$

where C *is the circle centered at* $(0, 0)$ *and of radius* R, *taken in counterclockwise direction.*

Exercise 4 *Consider the vector field* $f : \mathbb{R}^2 \setminus \{(0, 0)\} \ni (x, y) \mapsto \big(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\big) \in \mathbb{R}^2$. *Compute the curve integral for the following curves:*

(i) The curve defined by the circle centered at $(0, 0)$ *and of radius* $\sqrt{2}$, *taken in counterclockwise direction, from* $(1, 1)$ *to* $(-\sqrt{2}, 0)$,

(ii) The curve defined by the unit circle centered at $(0, 0)$, *taken in counterclockwise direction,*

(iii) The curve defined by the circle centered at $(0, 0)$ *and of radius* $r > 0$, *taken in counterclockwise direction.*