
Homework 4

Exercise 1 (i) Compute the Taylor expansion around $(0, 0)$ and up to the second order of the function

$$\mathbb{R}^2 \ni (x, y) \mapsto e^{x^2+xy+y^2} \in \mathbb{R}.$$

(ii) Compute the Taylor expansion around $(0, 0)$ up to the third order of the function

$$\mathbb{R}^2 \ni (x, y) \mapsto e^{x+y} \in \mathbb{R}.$$

By fixing then $x = y = 1/2$ in the polynomial you have obtained, what can you say about the number e ?

Exercise 2 Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = 2x^3 + 6xy - 3y^2 + 2$ for any $(x, y) \in \mathbb{R}^2$.

(i) Determine the local extrema of f ,

(ii) Does f possess global extrema ?

(iii) Consider the segment L defined by

$$L = \{(x, y) \in \mathbb{R}^2 \mid -2 \leq x \leq 0, y = x + 1\}$$

and determine the global extrema of f restricted to L . Where are these extrema located ?

Exercise 3 Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = xy e^{-\frac{1}{2}(x^2+y^2)}$.

(i) Study the local extrema of f (you can use the symmetries of this function),

(ii) Show that $f(x, y) \rightarrow 0$ as $\|(x, y)\| \rightarrow \infty$,

(iii) Deduce that there exist some global extrema and compute them.