Appendix 8

Summary Chap. IV and V

Framework: $\underline{X} = (X_1, \dots, X_n)$ a random sample, with $X_j \sim f(\cdot|\theta)$ (pmf or pdf), $f_{\underline{X}}(\cdot|\theta)$ is the joint pmf or pdf.

IV : Point estimation :

Aim: infer an *estimator* (= expression) for θ based on $\underline{X}$, or an *estimate* (= value) for θ based on $\underline{x}$ (= values obtained for $\underline{X}$ after the experiment).

Estimators or estimates are *statistics* $W(\underline{X})$ or $W(\underline{x})$.

Different methods (for finding $W(\underline{X})$ or $W(\underline{x})$):

- Method of moments
- Maximum likelihood estimator (MLE)
- Bayes estimator.

↗ there are other methods ...

Evaluate the estimator $W = W(\underline{X})$:

- Mean square error (MSE)

  ⇝ Compute $E_\theta((W-\theta)^2) = Var_\theta(W) + Bias_\theta(W)^2$

  $= (E_\theta(W-\theta))^2$

  and look for W minimizing this expression.

  ⇝ Best estimator or best unbiased estimator.

  ↑ when $Bias_\theta(W) = 0$

- Loss function optimality :

  ⇝ Replace $s \mapsto (s-\theta)^2$ by other functions.

V : Hypothesis testing :

Aim : Define 2 complementary hypotheses H_0 and H_1, and test them.

↑ Example : $\theta \in \Theta_0$

Different tests (for accepting or rejecting H_0)

- Likelihood ration tests (LRT)
- Bayesian tests
- Union-intersection and intersection-union tests.

Evaluate the test (which tests $H_0 : \theta \in \Theta_0$):

Truth \ Decision:	Accept H_0	Reject H_0
H_0	OK	Type I error
H_1	Type II error	OK

← based on experiments and the test

We would like to minimize Type I and Type II errors.

⇝ Power function β (for estimating Type I error)

⇝ Minimize β on Θ_0 and maximize β on Θ_0^c.

⇝ Size α-test and level α-test.

⇝ Uniformly most powerful (UMP) level α-test

A more continuous evaluation : p-value.

Additional work :

- p-value and conditioning p 399 of [CB] + Example 8.3.30
- Section 8.3.5 on loss function optimality