Let G be a $\overset{\text{finite}}{\text{group}}$ and let $\overbrace{\xi, \eta, \zeta}^{\to \text{equivalence classes}}$ denote the elements of $M(G)$ (Abelian)

Let $\{\xi_1, \cdots, \xi_{\tilde{n}}\} \subset M(G)$ be ~~the~~ a minimal generating set of $M(G)$, which means

$\forall \xi \in M(G) : \xi = \xi_1^{n_1} \xi_2^{n_2} \cdots \xi_{\tilde{n}}^{n_{\tilde{n}}} \quad \exists n_1, \cdots, n_{\tilde{n}} \in \mathbb{N}$

and one cannot do it with less elements.

For each ξ_s one sets K_s for the K(order) defind by Prop $(*)$ for ξ_s.

and set $S_s := e^{2\pi i / k_s}$

In addition $\exists \underline{\omega_s} \in \xi_s$ s.t. $\omega_s(a,b) = \varepsilon_s^{\,n(a,b)} \quad \exists n : G \times G \mapsto \{0, 1, \cdots, k_s - 1\}$

By the 2-cocycles relations, one has

$n(a,e) = 0 = n(e,b)$

$n(a,b) + n(ab,c) = n(a,bc) + n(b,c)$

Finally we set $\phi : G \times G \mapsto M(G)$ by

$$\phi(a,b) = \prod_{s=1}^{\tilde{n}} \xi_s^{n_s(a,b)} \in M(G)$$

Thm. Let G be a finite group and $M(G)$ its Schur multiplier

Let $\tilde{G} := \{(\xi, a) \in M(G) \times G\}$ with the product

$(\xi, a) \cdot (\eta, b) = (\xi \eta \phi(a,b), ab)$. $\top$

Then

1) $\tilde{G}$ with the above multiplication is a group ;

~~2)~~ $\tilde{G}_0 := \{(\xi, e) \mid \xi \in M(G)\} \lhd \tilde{G}$, and $\tilde{G}/\tilde{G}_0 \simeq G$. Proof as exercise

2) If $(U, \mathcal{H})$ is a proj. rep. of G then

$\exists p : G \mapsto \mathbb{C}^*$ and

$(\tilde{U}, \mathcal{H})$ a linear rep. of G s.t. $U(a) = p(a) \tilde{U}([1], a)$ Proof as exercise

This defines an equivalent proj. rep