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No.

special mathmatic lecture

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1, check in examples

(1.1) Which are subgroups?

$(\mathbb{Z}, +)$ is a subgroup of $(\mathbb{R}, +)$

since $\mathbb{R} \subset \mathbb{C}$,

$(GL(n,\mathbb{R}), \times)$	〃	$(GL(n,\mathbb{C}), \times)$			
$(SL(n,\mathbb{R}), \times)$	〃	$(SL(n,\mathbb{C}), \times)$			
$(O(n), \times)$	〃	$(U(n), \times)$			
$(SO(n), \times)$	〃	$(SU(n), \times)$			
$(SL(n,\mathbb{R}), \times)$	〃	$(GL(n,\mathbb{R}), \times)$	, $(SL(n,\mathbb{C}), \times)$	is a subgroup of	$(GL(n,\mathbb{C}), \times)$
$(O(n), \times)$	〃	$(GL(n,\mathbb{R}), \times)$	, $(U(n), \times)$	〃	$(GL(n,\mathbb{C}), \times)$
$(SO(n), \times)$	〃	$(GL(n,\mathbb{R}), \times)$	, $(SU(n), \times)$	〃	$(GL(n,\mathbb{C}), \times)$
$(SO(n), \times)$	〃	$(O(n), \times)$	, $(SU(n), \times)$	〃	$(U(n), \times)$
$(SO(n), \times)$	〃	$(SL(n,\mathbb{R}), \times)$	, $(SU(n), \times)$	〃	$(SL(n,\mathbb{C}), \times)$

(1.2) Does C_n contain subgroups?

If n is a prime number, it doesn't.

If $n = pq$ ($p, q \in \mathbb{Z}^+$, $p, q \neq 1$) then

a set $\{e, a^p, a^{2p}, a^{3p}, \ldots, a^{p(q-1)}\}$ and a rule ($a^i \cdot a^j = a^{(i+j) \bmod n}$)

is a subgroup of C_n.

2, Left coset, Right coset,

(2.1) proof of ${}_{G_0}[a] = aG_0$

(2.1.1) At first, I prove ${}_{G_0}[a] \subset aG_0$

for $\forall b$, $b \in {}_{G_0}[a] \Longleftrightarrow a^{-1}b \in G_0$

multiply both sides of $a^{-1}b \in G_0$ by a from left.

$\rightarrow$ $aa^{-1}b \in aG_0$

$b = eb \in aG_0$

so, for $\forall b \in {}_{G_0}[a]$, $b \in aG_0 \longrightarrow {}_{G_0}[a] \subset aG_0$

(2.1.2) Second, I prove ${}_{G_0}[a] \supset aG_0$

for $\forall c \in aG_0$, $c = ad \mid d \in G_0$,

multiply both sides of $c = ad$ by a^{-1} from left

$\rightarrow$ $a^{-1}c = a^{-1}ad = ed = d$.

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since $d \in G_0$ and $a^{-1}c = d$, $a \sim c$,
so for $\forall c \in aG_0$, $c \in G_0[a]$ $\rightarrow$ $G_0[a] \supset aG_0$

(2.1.3) From (2.1.1) and (2.1.2),
since $G_0[a] \subset aG_0$ and $G_0[a] \supset aG_0$,
$\underline{G_0[a] = aG_0}$ $\square$

(2.2) proof of $[a]_{G_0} = G_0 a$

(2.2.1) I prove $[a]_{G_0} \subset G_0 a$
for $\forall b$, $b \in [a]_{G_0} \Leftrightarrow ba^{-1} \in G_0$
multiply both sides of $ba^{-1} \in G_0$ by a from right.
$\rightarrow$ $ba^{-1}a = be = b \in G_0 a$
so, for $\forall b \in [a]_{G_0}$, $b \in G_0 a$, $\longrightarrow$ $[a]_{G_0} \subset G_0 a$

(2.2.2) I prove $[a]_{G_0} \supset G_0 a$
for $\forall c \in G_0 a$, $c = da \mid d \in G_0$,
multiply both sides of $c = da$ by a^{-1} from right.
$\rightarrow$ $ca^{-1} = daa^{-1} = de = d \in G_0$ $\rightarrow$ $ca^{-1} \in G_0$
so $a \overset{r}{\sim} c$. and $c \in [a]_{G_0}$
for $\forall c \in G_0 a$, $c \in [a]_{G_0}$ $\longrightarrow$ $[a]_{G_0} \supset G_0 a$

(2.2.3) From (2.2.1) and (2.2.2)
since $[a]_{G_0} \subset G_0 a$ and $[a]_{G_0} \supset G_0 a$,
$\underline{[a]_{G_0} = G_0 a}$ $\square$

3, homomorphism, (page 4)
Prop. Let ϕ be a group homomorphism from G to G'
(2) $\phi(e_G) = e_{G'}$ and $\phi(a^{-1}{}_G) = (\phi(a))^{-1}{}_{G'}$ $\leftarrow$ proof of these.

i.e. of G   i.e. of G'

(3.1) proof of $\phi(e_G) = e_{G'}$

calculation in G   calculation in G'

From the definition of homomorphism, $\phi(ab) = \phi(a)\phi(b)$

When $a=e$, (identity element of G)

$\phi(e \cdot b) = \phi(e)\phi(b)$

$\phi(b) = \phi(e)\phi(b)$

When $b=e$ (=)

$\phi(a \cdot e) = \phi(a)\phi(e)$

$\phi(a) = \phi(a)\phi(e)$

so, since $\phi(a) = \phi(a)\phi(e) = \phi(e)\phi(a)$,

$\phi(e)$ is identity element in G'

(3.2) proof of $\phi(a^{-1_G}) = (\phi(a))^{-1_{G'}}$

From $\phi(a \cdot b) = \phi(a)\phi(b)$.

when $b=a^{-1}$,

$\phi(a \cdot a^{-1}) = \phi(a)\phi(a^{-1})$

$\phi(e) = \phi(a)\phi(a^{-1})$

multiply both side by $(\phi(a))^{-1}$ from left.

$\rightarrow (\phi(a))^{-1}\phi(e) = (\phi(a))^{-1}\phi(a) \cdot \phi(a^{-1})$

$\rightarrow (\phi(a))^{-1} = \phi(e)\phi(a^{-1})$

$\rightarrow \phi(a^{-1}) = (\phi(a))^{-1}$ □