

Thm 2. Let \mathcal{I} be an open interval in \mathbb{R}

Suppose that :

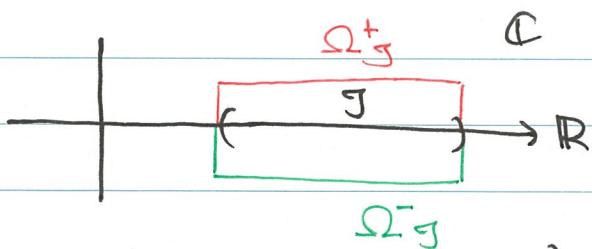
1) G_0 is H_0 -smooth on \mathcal{I} ,

2) $(G_0 R_0(z) G^*)^l \in K(\mathcal{I})$ for some $l \in \mathbb{N}$,

3) The map $z \mapsto G_0 R_0(z) G^*$

and $z \mapsto G R_0(z) G^*$ are holomorphic

in $\Omega_{\mathcal{I}}^{\pm}$ and extend continuously to \mathcal{I}



Then $W_{\pm}(H, H_0, E_{ac}^{H_0}(\mathcal{I}))$ exist and

are complete $\left(\Leftrightarrow W_{\pm}(H_0, H, E_{ac}^H(\mathcal{I})) \text{ exist} \right)$

Remark : There is no smallness condition in the

assumptions, but the result is only on $\mathfrak{H}_{ac}(H_0)$
or $\mathfrak{H}_{ac}(H)$.

Proof.

Recall that N was introduced in the abstract thm and N is closed and of measure 0 (in J). It implies that $\exists \{J_n\}_{n \in \mathbb{N}}$ subintervals in J , J_n open, with

$$J \setminus N = \bigsqcup_n J_n \quad \text{and} \quad |J| = |\bigsqcup_n J_n|.$$

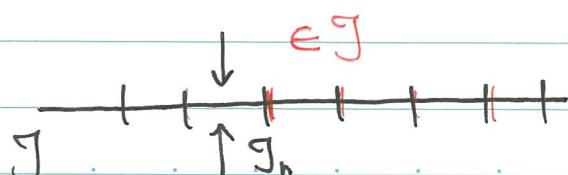
We shall now use the abstract thm \circledast

with $A(z) = -B_0(z) = G_0 R_0(z) G^*$.

As a consequence of thm \circledast , the map

$$z \mapsto (1 - G_0 R_0(z) G^*)^{-1}$$

is norm continuous when z approaches any point in J_n



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$$\Rightarrow z \mapsto (1 - G_0 R_0(z) G^*)^{-1} \text{ is}$$

locally norm continuous on \mathbb{J}_n

$$\Rightarrow z \mapsto GR(z)G^* = GR(z)G^*(1 - G_0 R_0(z)G^*)^{-1}$$

is locally norm continuous on \mathbb{J}_n

$\Rightarrow G$ is locally H -smooth on \mathbb{J}_n

Since G_0 is also locally H_0 -smooth

on \mathbb{J} , it implies that $W_{\pm}(H, H_0, E^{H_0}(\mathbb{J}_n))$

exist and are complete.

Since $N = \mathbb{J} \setminus \bigcup_n \mathbb{J}_n$ is of Lebesgue measure 0

one has $E_{ac}^H(\mathbb{J}) = E_{ac}^H\left(\bigcup_n \mathbb{J}_n\right) = \bigoplus_n E_{ac}^H(\mathbb{J}_n)$

\uparrow
is not true for E_{sc}^H or E_p^H .

and the same for H_0 instead of H .

One infers that $W_{\pm}(H, H_0, E_{ac}^{H_0}(\mathbb{J}))$

exist and are complete.

A few more information for thm 1 and
thm 2.

Proposition Suppose that assumptions of
thm 1 are satisfied, and suppose that
 $\text{Ker}(G) = \{0\}$. Then the spectrum of
 H on \mathcal{J} is purely a.c.

Proof

In the proof of thm 1, we get that G

is H -smooth on \mathcal{J} . It follows from

the condition 5 on H -smoothness that

$$\langle G^*f, E^H(\Lambda)G^*f \rangle \leq \pi_5^2 |\Lambda| \|f\|$$

for any interval $\underline{\Lambda} \subset \mathcal{J}$.

$$\Rightarrow G^*f \in \mathcal{H}_{ac}(H) \cap E^H(\mathcal{J})\mathcal{H}$$

$$\Leftrightarrow \text{Ran}(G^*) \subset \mathcal{H}_{\text{ac}}(H) \cap E^H(J)\mathcal{H}$$

$$\Leftrightarrow \text{Ran}(GE^H(J)) \subset \mathcal{H}_{\text{ac}}(H)$$

Recall that $\mathcal{H} = \underbrace{\text{Ran}(G_J^*)}_{\subset \mathcal{H}_{\text{ac}}(H)} \oplus \underbrace{\text{Ker}(G_J)}_{\supset \mathcal{H}_c(H)} := \mathcal{H}_{\text{sc}}(H) \oplus \mathcal{H}_p(H)$

We infer $\mathcal{H}_c(H) \subset \underbrace{\text{Ker}(G_J)}$

$$= \text{Ker}(GE^H(J)) = \text{Ker}(G) \Big|_{E^H(J)\mathcal{H}}$$

$$= \{0\},$$

Thus we get $\mathcal{H}_c(H) \cap E^H(J)\mathcal{H} = \underline{\{0\}}$.



The proof is valid in $E^H(J)\mathcal{H}$. □

$\vdash \vdash \vdash \vdash \vdash \vdash$ (localize to subintervals)

Proposition Suppose that assumptions of

Thm 2 are satisfied, and recall that

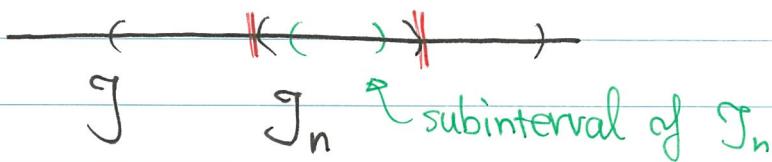
$$\mathcal{N} = \{ \lambda \in \overline{J} \mid 1 \in \sigma(B_0(\lambda \pm i0)) \}$$

Then, if $\ker(G) = \{0\}$ then H is
purely a.c. on $\mathbb{R} \setminus N$.

Idea of the proof:

A local version of the previous proof.

$\in N$



in any strict subinterval of J_n .

What about N ?

With stronger assumptions one gets

that $N = \sigma_p(H) \cap J$, and the

multiplicities of the eigenvalue $\lambda \in \sigma_p(H)$

and 1 in $\sigma(B_0(\lambda \pm i0))$ coincide.

$$B_0(z) := G_0 R_0(z) G^*$$