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No.

Lecture 4 final range projection (corrected mistakes in my previous report.)

Consider H, H_0, E and $W_\pm(H, H_0, E)$ and set $F := W_\pm W_\pm^*$

① $F e^{-itH} = e^{-itH} F$

② $W_\pm(H_0, H, F)$ exist, and $W_\pm(H_0, H, F) = W_\pm(H, H_0, E)^*$

proof

① $W^* e^{-itH} = W^* (e^{itH})^* = (e^{itH} W)^*$

$= (W e^{itH_0})^*$ ($\because$ intertwining property)

$= e^{-itH_0} W^*$

$\therefore F e^{-itH} = W_\pm W_\pm^* e^{-itH} = W_\pm e^{-itH_0} W_\pm^*$

$= e^{-itH} W_\pm W_\pm^*$ □

② For all $g \in F\mathcal{H}$, there exists $f \in E\mathcal{H}$ such that $g = W_\pm f$.

Then
$$0 = \lim_{t\to\pm\infty} \| e^{itH} e^{-itH_0} f - g \|$$
$$= \lim_{t\to\pm\infty} \| f - e^{itH_0} e^{-itH} g \|$$

$\Rightarrow \operatorname*{s-lim}_{t\to\pm\infty} e^{itH_0} e^{-itH} g$ exists for all $g \in F\mathcal{H}$

$\Rightarrow \operatorname*{s-lim}_{t\to\pm\infty} e^{itH_0} e^{-itH} F = W(H_0, H, F)$ exists

$$\operatorname*{s-lim}_{t\to\pm\infty} e^{itH_0} e^{-itH} F = \operatorname*{s-lim}_{t\to\pm\infty} e^{itH_0} e^{-itH} W_\pm(H, H_0, E) W_\pm(H, H_0, E)^*$$
$$= \operatorname*{s-lim}_{t\to\pm\infty} e^{itH_0} e^{-itH} e^{itH} e^{-itH_0} E W_\pm(H, H_0, E)^*$$
$$= \operatorname*{s-lim}_{t\to\pm\infty} E W_\pm(H, H_0, E)^*$$
$$= \operatorname*{s-lim}_{t\to\pm\infty} (W_\pm(H, H_0, E) E)^* \quad (\because E^* = E)$$
$$= \operatorname*{s-lim}_{t\to\pm\infty} (e^{itH} e^{-itH_0} E^2)^*$$
$$= \operatorname*{s-lim}_{t\to\pm\infty} (e^{itH} e^{-itH_0} E)^* \quad (\because E^2 = E)$$
$$= W_\pm^*(H, H_0, E)$$ □