Homework 8

Exercise 1 *Consider the vector field* $F:\mathbb{R}^2\setminus\{(0,0)\}\to\mathbb{R}^2$ *defined for* $(x,y)\neq(0,0)$ *by*

$$F(x,y)=\Big(\frac{-y}{x^2+y^2},\frac{x}{x^2+y^2}\Big)$$

(i) Represent graphically this vector field (you can use polar coordinates),

(ii) Can you find a potential function for this vector field, and if so exhibit it.

Exercise 2 *Consider the map*

$$f:\mathbb{R}^2\ni\ (x,y)\mapsto x^3-2xy+2y^2-1\ \in\mathbb{R}.$$

(i) Show that the implicit function theorem can be applied at the point $(1,1)\in\mathbb{R}^2$*,*

(ii) Compute the tangent at the point $(1,1)$ *of the curve of equation* $f(x,y)=0$*, and determine the position of this curve with respect to the tangent line at this point.*

Exercise 3 *Consider the map*

$$f:\mathbb{R}^3\ni\ (x,y,z)\mapsto x^2-xy^3-y^2z+z^3\ \in\mathbb{R}.$$

(i) Show that the implicit function theorem can be applied at the point $(1,1,1)$*. We shall call* ϕ *the implicit function defined on* $B_\varepsilon\big((1,1)\big)$ *for some* $\varepsilon>0$ *and which expresses* z *in terms of* x,y *for* z *near the value* 1*,*

(ii) Determine the equation of the plane tangent to the surface defined by $f(x,y,z)=0$ *at the point* $(1,1,1)$*.*