Nagoya University, G30 program Spring 2016

Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

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Homework 9

Exercise 1 *Let $f:\mathbb{R}^n\to\mathbb{R}$ be a continuous function, and let $c:[a,b]\to\mathbb{R}^n$ be a parametric curve of class C^1. What kind of integral can you define with these objects such that the result does not depend on the parametrization of the curve ? Can you prove your statement ?*

Exercise 2 *Compute the following integrals:*

$$\iint_\Omega x^2 y\,\mathrm{d}x\,\mathrm{d}y \quad \textit{with} \quad \Omega=[1,2]\times[-3,4],$$

$$\iiint_\Omega \sin(x)y\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \quad \textit{with} \quad \Omega=[0,\pi]\times[0,1]\times[0,2]$$

Exercise 3 *1) Compute the integral $\iint_\Omega \mathrm{e}^{x+y}\,\mathrm{d}x\,\mathrm{d}y$ with Ω the subset of $\mathbb{R}^2$ defined by* $\{(x,y)\in\mathbb{R}^2 \mid |x|+|y|\leq 1\}$,

2) Compute the integral $\iint_\Omega (x-y)\,\mathrm{d}x\,\mathrm{d}y$ with Ω the subset of $\mathbb{R}^2$ defined by the three lines of equation $x=0$, $y=x+2$, and $y=-x$,

3) Compute the integral $\iint_\Omega xy\,\mathrm{d}x\,\mathrm{d}y$ with Ω the subset of $\mathbb{R}^2$ defined by the two functions of equation $y=x^2$ and $y=x^3$.

Exercise 4 *Compute the integral $\iiint_\Omega (x+y+z)^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ with Ω the subset of $\mathbb{R}^3$ defined by the four planes of equation $x=0$, $y=0$, $z=0$, and $x+y+z=1$.*