Nagoya University, G30 program Spring 2016
Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

---

Homework 4

Exercise 1 *Consider the function* $f : \mathbb{R}^2 \to \mathbb{R}$ *defined by* $f(x, y) = 2x^3 + 6xy - 3y^2 + 2$ *for any* $(x, y) \in \mathbb{R}^2$.

(i) Determine the local extrema of f,

(ii) Does f *possess global extrema ?*

(iii) Consider the segment L *defined by*

$$L = \big\{(x, y) \in \mathbb{R}^2 \mid -2 \leq x \leq 0,\ y = x + 1\big\}$$

and determine the global extrema of f *restricted to* L*. Where are these extrema located ?*

Exercise 2 *Consider the function* $f : \mathbb{R}^2 \to \mathbb{R}$ *defined by* $f(x, y) = x\,y\,\mathrm{e}^{-\frac{1}{2}(x^2+y^2)}$.

(i) Study the local extrema of f *(you can use the symmetries of this function),*

(ii) Show that $f(x, y) \to 0$ *as* $\|(x, y)\| \to \infty$,

(iii) Deduce that there exist some global extrema and compute them.

Exercise 3 *Draw the unit ball of* $\mathbb{R}^2$ *with the three norms* $\|\cdot\|_2$, $\|\cdot\|_1$ *and* $\|\cdot\|_\infty$.

Exercise 4 *Consider the functions* $\varphi : \mathbb{R} \to \mathbb{R}^2$ *defined by* $\varphi(t) := \binom{\cos(t)}{t^2}$, *and* $f : \mathbb{R}^2 \to \mathbb{R}$ *defined by* $f(x, y) := \mathrm{e}^{3x+2y}$. *We consider the composition of these two functions, namely* $F : \mathbb{R} \to \mathbb{R}$ *given by* $F = f \circ \varphi$. *Compute the derivative of this function by two different methods: once by a direct computation, and once as the derivative of a composed function (chain rule).*

Exercise 5 *Let us consider* $f : \mathbb{R}^3 \to \mathbb{R}$ *defined by* $f(x, y, z) = \mathrm{e}^{xy}\cos(z)$ *for any* $(x, y, z) \in \mathbb{R}^3$. *Assume also that* $x = tu$, $y = \sin(tu)$ *and* $z = u^2$ *for some* $t, u \in \mathbb{R}$. *By setting*

$$F(t, u) := f\big(tu, \sin(tu), u^2\big)$$

Compute the derivative $\partial_2 F \equiv \partial_u F$ *by two different methods: once by a direct computation, and once as the derivative of a composed function (chain rule). For the second method, note that one often sees expressions like*

$$\frac{\partial F}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial u}.$$