Nagoya University, G30 program Spring 2016
Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

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Homework 1

Exercise 1 *Let* $f : (a, b) \to \mathbb{R}^d$ *be a parametric curve of class* C^1*, and let* $\varphi : (c, d) \to (a, b)$ *be of class* C^1 *and strictly increasing. Show that* $L_f = L_{f\circ\varphi}$*, or in other words show that the length of the curve does not depend on the parametrization.*

Exercise 2 *Write a parametric equation for the tangent line at any point of the curve given by*

$$g : \mathbb{R} \ni t \mapsto \big(\mathrm{e}^{3t}, \mathrm{e}^{-3t}, t, 1\big) \in \mathbb{R}^4.$$

Exercise 3 *Consider the spiral in* $\mathbb{R}^3$ *defined by the function*

$$f : \mathbb{R} \ni t \mapsto \big(\cos(t), \sin(t), t\big) \in \mathbb{R}^3.$$

Determine the equation of the plane perpendicular to the spiral for any $t \in \mathbb{R}$*.*

Exercise 4 *Find the length of the spiral of the previous exercise between* $t = 0$ *and* $t = 1$*.*

Exercise 5 *Consider the parametric curve given by*

$$c : \mathbb{R} \ni t \mapsto \big(\mathrm{e}^t \cos(t), \mathrm{e}^t \sin(t)\big)$$

Show that the tangent vector to the curve makes a constant angle with the position vector, *i.e.* with the vector $c(\cdot)$.

Exercise 6 *Consider a parametric curve* $f : \mathbb{R} \to \mathbb{R}^d$ *of class* C^2*, and us call* the osculating plane at t *the plane passing by* $f(t)$ *and defined by the two vectors* $f'(t)$ *and* $f''(t)$*. Obviously this plane is well defined only if these two vectors are not parallel.*

(i) Determine the osculating plane at any t *for the spiral defined in Exercise 3.*

(i) More generally, for any parametric curve f *of class* C^2 *and for any diffeomorphism* φ *of class* C^2*, show that the osculating plane defined by* f *or by the new parametric curve* $f \circ \varphi$ **at the same point** *are equal.*