**Nagoya University, G30 program** Spring 2015
**Math tutorial II, linear algebra** Instructor : Serge Richard
**Quiz III** Name : ---------------------------------------

---

**Exercise 1** *Circle the correct statement(s), without justification:*

1. *The determinant of a matrix with only* 0 *on its diagonal is equal to* 0*,*

   No, $\mathrm{Det}\begin{pmatrix}0&1\\1&0\end{pmatrix} = -1$.

2. *If* 0 *is an eigenvalue of* $\mathcal{A}\in M_n(\mathbb{R})$*, then* $\mathcal{A}$ *can not be invertible,*

   Since $\mathcal{A}X = 0$ for some $X\in\mathbb{R}^n$, $X\neq 0$, one has $\mathrm{Ker}(L_\mathcal{A})\neq\{0\}$ $\Rightarrow$ $L_\mathcal{A}$ is not invertible $\Rightarrow$ $\mathcal{A}$ not invertible.

3. *If two* $n\times n$ *matrices* $\mathcal{A}$ *and* $\mathcal{B}$ *are similar, then the equality* $\mathsf{Det}(\mathcal{A}) = \mathsf{Det}(\mathcal{B})$ *must hold,*

   If $\mathcal{B} = \mathcal{U}\mathcal{A}\mathcal{U}^{-1}$ then $\mathrm{Det}(\mathcal{B}) = \mathrm{Det}(\mathcal{U})\,\mathrm{Det}(\mathcal{A})\,\mathrm{Det}(\mathcal{U}^{-1})$
   $= \mathrm{Det}(\mathcal{A})$.

4. *The determinant of a symmetric matrix can be equal to* 0*,*

   $\begin{pmatrix}1&1\\1&1\end{pmatrix}$ is symmetric and its determinant is 0.

5. *If* $\mathcal{A},\mathcal{B}\in M_n(\mathbb{R})$*, then* $\mathsf{Det}(\mathcal{A}+\mathcal{B}) = \mathsf{Det}(\mathcal{A}) + \mathsf{Det}(\mathcal{B})$*,*

   No, if $\mathcal{A} = \begin{pmatrix}1&0\\0&0\end{pmatrix}$, $\mathcal{B} = \begin{pmatrix}0&0\\0&1\end{pmatrix}$, then $\mathrm{Det}(\mathcal{A}+\mathcal{B}) = 1$ but $\mathrm{Det}(\mathcal{A}) = \mathrm{Det}(\mathcal{B}) = 0$.

6. *For any* $\mathcal{A}\in M_n(\mathbb{R})$*, one has* $\mathsf{Det}\,(\mathcal{A}\,{}^t\!\mathcal{A}) = \mathsf{Det}\,({}^t\!\mathcal{A}\,\mathcal{A})$*,*

   Since $\mathrm{Det}({}^t\!\mathcal{A}\,\mathcal{A}) = \mathrm{Det}({}^t\!\mathcal{A})\,\mathrm{Det}(\mathcal{A}) = \mathrm{Det}(\mathcal{A})\,\mathrm{Det}({}^t\!\mathcal{A})$
   $= \mathrm{Det}(\mathcal{A}\,{}^t\!\mathcal{A})$.

7. *If* $\mathcal{A}\in M_3(\mathbb{R})$*, then* $\mathsf{Det}(-\mathcal{A}) = \mathsf{Det}(\mathcal{A})$*,*

   No, $\mathrm{Det}\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{pmatrix} = -1 \neq 1 = \mathrm{Det}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$.

8. *If* $\mathcal{A}\in M_n(\mathbb{R})$ *and* $\mathcal{A}^2 = \mathbf{1}_n$*, then* $\mathsf{Det}(\mathcal{A}) = 1$*,*

   No, if $\mathcal{A} = \begin{pmatrix}0&1\\1&0\end{pmatrix}$ then $\mathcal{A}^2 = \mathbb{1}_2$ but $\mathrm{Det}(\mathcal{A}) = -1$.

9. *If all entries of a* $3\times 3$ *matrix are* 3*, its determinant must be* $3^3 = 27$*,*

   No, its determinant is 0.

10. *The sum of two invertible matrices is invertible.*

    No, $\mathbb{1}_n$ and $-\mathbb{1}_n$ are invertible, but $\mathbb{1}_n + (-\mathbb{1}_n) = 0_n$ and $0_n$ is not invertible.