Nagoya University, G30 program Spring 2015
Linear algebra II Instructor : Serge Richard
Quiz II Name : ____________________

Exercise 1 *Complete the following sentence:*
A map $F : V \to W$ between two sets V and W is surjective if... for any $Y \in W$, there exits at least one $X \in V$ with $F(X) = Y$.

Exercise 2 *Circle the correct statement(s), <u>without justification</u>. If V is a real vector space with a scalar product $\langle \cdot, \cdot \rangle$ and if X, Y are arbitrary elements of V, then:*

1. $\langle X, Y \rangle \geq 0$,
2. $\|X + Y\|^2 = \|X\|^2 + \|Y\|^2$,
3. $\langle X, Y \rangle \leq \|X\| \, \|Y\|$,
4. $\|X + Y\|^2 \leq (\|X\| + \|Y\|)^2$.

Exercise 3 *Let $V_1 := \frac{1}{\sqrt{2}} \binom{1}{-1}$. Find a vector V_2 of $\mathbb{R}^2$ such that $\{V_1, V_2\}$ define a orthonormal basis of $\mathbb{R}^2$.*

For example $V_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ since $\|V_2\| = 1$ and $V_1 \cdot V_2 = 0$.

Exercise 4 *Let $\mathcal{A} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ with $a, b \in \mathbb{R}$. Under which conditions on a and b does the following equivalence holds:*

$$^T X \mathcal{A} X = 0 \quad \textit{if and only if} \quad X = \binom{0}{0}.$$

Circle the letter(s) which correspond to a correct answer (no need to justify).

a) $a + b = 0$,

b) $ab = 0$,

c) $a > 0$ and $b > 0$,

d) $a < 0$ and $b > 0$.

because $^T X \mathcal{A} X = a x_1^2 + b x_2^2$ if $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, and $a x_1^2 + b x_2^2 \neq 0$ for all $\binom{x_1}{x_2} \neq \binom{0}{0}$ if and only if $(a, b) \neq (0, 0)$ and a, b are of the same sign.