Calculus II

Quiz 4

Name: ME

Explain your solution process clearly.
Write legible.

1. (5 points) Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a differentiable function. Suppose

$$f(1,2) = 3, \ f_x(1,2) = 2 \text{ and } f_y(1,2) = -1$$

Let $g(t) = (3\cos(t) - 2 + 2t, \sin^2(t) + 2)$. Find $\frac{d}{dt}(f \circ g)(0)$.

$$g(0) = (3\cos(0) - 2, \sin^2(0) + 2) = (3-2, 2) = \underline{(1,2)}$$

$$\frac{d}{dt}(f \circ g)(0) = f_x(g(0)) \cdot g_1'(0) + f_y(g(0)) \cdot g_2'(0), \text{ where}$$

$$g_1(t) = 3\cos t - 2 + 2t \Rightarrow g_1'(t) = -3\sin t + 2 \Rightarrow g_1'(0) = 2$$

$$g_2(t) = \sin^2(t) + 2 \Rightarrow g_2'(t) = 2\sin(t) \cdot \cos(t) \Rightarrow g_2'(0) = 0$$

$$\Rightarrow \underline{\frac{d}{dt}(f \circ g)(0) = 4}$$

2. (5 points) If $w = f\left(\frac{xy}{x^2+y^2}\right)$ is a differentiable function of $u = \frac{xy}{x^2+y^2}$, show that

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} = 0$$

$$w_x = f'\left(\frac{xy}{x^2+y^2}\right) \cdot \left(\frac{y(x^2+y^2) - xy \cdot 2x}{(x^2+y^2)^2}\right) = f'\left(\frac{xy}{x^2+y^2}\right) \cdot \frac{-yx^2+y^3}{(x^2+y^2)^2}$$

$$w_y = f'\left(\frac{xy}{x^2+y^2}\right) \cdot \frac{-xy^2+x^3}{(x^2+y^2)^2}$$

$$\Rightarrow x \cdot w_x + y \cdot w_y = f'\left(\frac{xy}{x^2+y^2}\right)\left(\frac{1}{(x^2+y^2)^2}\right)\left[-yx^3+xy^3-xy^3+x^3y\right]$$

$$= 0$$