Math Tutorial II (Calculus II)

Quiz 3

Name: ________________________

Explain your solution process clearly.
Write legible.

1. (5 points) Suppose that the surface of Mount Fuji is represented by a region D in the xy-plane such that the mountain's height (in meters) at the point (x, y) is given by the expression $3777 - e^{x^2y^4}$. If you are on the mountain at $(x, y) = (2, 5)$, in which direction should you hike

(a) so that the height decreases most rapidly? $h(x,y) = 3777 - e^{x^2y^4}$

$\Rightarrow \nabla h(x,y) = \left(-2xy^4 e^{x^2y^4}, -4x^2y^3 e^{x^2y^4}\right) \Rightarrow \nabla h(x,y) = -2xy^3 e^{x^2y^4}(y, 2x)$

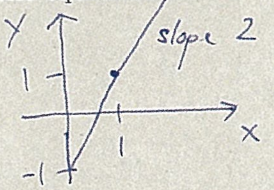
$\Rightarrow \nabla h(2,5) = -2 \cdot 2 \cdot 5^3 e^{2^2 5^4}(5,4)$;

Since $h(x,y)$ decreases most rapidly in $-\nabla h(x,y)$ one should hike in direction of ~~[illegible]~~ $(5,4)$.

(b) so that the height remains constant?

We are looking for a unit vector $\vec{v}$ such that $(D_{\vec{v}} h)(2,5) = 0$. Since $(D_{\vec{v}} h)(2,5) = -(v_1, v_2) \cdot 4 \cdot 5^3 e^{4 \cdot 5^4}(5,4)$, it follows that $\vec{v}$ may be chosen as $(-4,5)/_{\dots}$

2. (5 points) The surface $z = 3x^2 + \frac{1}{6}x^3 - \frac{1}{8}x^4 - 4y^2$ is intersected by the plane $2x - y = 1$. The resulting intersection is a curve of the surface. Find a set of parametric equations for the line tangent to this curve at the point $(1, 1, -\frac{23}{24})$.



Need $(D_{\vec{v}} z)(1,1)$ for $\vec{v} = (1,2)/\sqrt{5}$

$z_x(x,y) = 6x + \frac{1}{2}x^2 - \frac{1}{2}x^3 \Rightarrow z_x(1,1) = 6$

$z_y(x,y) = -8y \Rightarrow z_y(1,1) = -8$

$\Rightarrow (D_{\vec{v}} z)(1,1) = \frac{1}{\sqrt{5}}(1,2)\cdot(6,-8) = \frac{1}{\sqrt{5}}(6-16) = \frac{-10}{\sqrt{5}} = -\sqrt{20} = -2\sqrt{5}$

$\Rightarrow$ parametric equation for line tangent to the curve

$$\vec{l}(t) = \begin{pmatrix} 1 \\ 1 \\ -\frac{23}{24} \end{pmatrix} + t \begin{pmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \\ -2\sqrt{5} \end{pmatrix}$$