Calculus II

# Quiz 1

Name: ________________________

Explain your solution process clearly.
Write legible.

1. [10 points] Determine whether each of the following statements is true or false. If true, then prove the statement; if false give a counter example which shows that the statement is false.

(a) If $\vec{a}$ and $\vec{b}$ are two vectors in $\mathbb{R}^3$, and $k$ and $\ell$ are real numbers, then

$$(k-\ell)(\vec{a}+\vec{b}) = k\vec{a} - \ell\vec{a} + k\vec{b} - \ell\vec{b}.$$

TRUE

$$(k-\ell)(\vec{a}+\vec{b}) \underset{\substack{\uparrow\\ \text{distributivity}\\ \text{of vector addition}}}{=} (k-\ell)\vec{a} + (k-\ell)\vec{b} \underset{\substack{\uparrow\\ \text{distributivity of}\\ \text{scalar addition}}}{=} k\vec{a} - \ell\vec{a} + k\vec{b} - \ell\vec{b}$$

(b) For any vector $\vec{a}$ in $\mathbb{R}^3$ and scalar $k$, we have $\|k\vec{a}\| = k\|a\|$.

FALSE

Let $\vec{a} = (1,0,0)$, $k=-1$

$\Rightarrow \|k\vec{a}\| = \|(-1,0,0)\| = \sqrt{1^2+0^2+0^2} = 1$, but

$k\|\vec{a}\| = -1 \cdot \|(1,0,0)\| = -1 \cdot 1 = -1$ and $-1 \neq 1$.

(c) The dot product of two unit vectors is 1.

FALSE

$\vec{u}_1 = \begin{pmatrix}1\\0\end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix}0\\1\end{pmatrix}$ are both unit vectors but $\vec{u}_1 \circ \vec{u}_2 = \begin{pmatrix}0\\1\end{pmatrix} \circ \begin{pmatrix}1\\0\end{pmatrix} = 0 \neq 1$.

(d) Let $L_1$ and $L_2$ be two lines in $\mathbb{R}^2$. Then $L_1$ and $L_2$ either intersect in at least one point or are parallel.

TRUE

Let $L_1, L_2$ be given by $\begin{matrix} y = m_1 x + n_1 \\ y = m_2 x + n_2 \end{matrix}$. Then $L_1$ & $L_2$ intersect if

$m_1 x + n_1 = m_2 x + n_2$

$\Rightarrow (m_1 - m_2)x = n_2 - n_1$

$\Rightarrow$ either $m_1 = m_2$ (which means $L_1$ & $L_2$ are parallel)

or $m_1 \neq m_2$, then $\left(\frac{n_2-n_1}{m_1-m_2}, m_1\left(\frac{n_2-n_1}{m_1-m_2}\right)+n_1\right)$ is a point of intersection of the lines.

(e) Let $L_1$ and $L_2$ be two lines in $\mathbb{R}^3$. Then $L_1$ and $L_2$ either intersect in at least one point or are parallel.

FALSE.

Let $L_1$ be given by $\begin{cases} x(t) = t \\ y(t) = 0 \\ z(t) = 0 \end{cases}$ and $L_2$ by $\begin{cases} x(t) = 0 \\ y(t) = t \\ z(t) = 1 \end{cases}$

$\Rightarrow L_1 \parallel \begin{pmatrix}1\\0\\0\end{pmatrix}$, $L_2 \parallel \begin{pmatrix}0\\1\\0\end{pmatrix} \Rightarrow L_1 \nparallel L_2$ and

there is no solution to: $\begin{cases} t = 0 \\ 0 = s \\ 0 = 1 \end{cases} \Rightarrow L_1$ & $L_2$ don't intersect.