Problem Set 9 - Math Tutorial Calculus II

1. The surface $z = 3x^2 + \frac{1}{6}x^3 - \frac{1}{8}x^4 - 4y^2$ is intersected by the plane $2x - y = 1$. The resulting intersection is a curve of the surface. Find a set of parametric equations for the line tangent to this curve at the point $(1, 1, -\frac{23}{24})$.

2. Show that the plane tangent to a sphere at a point P on the sphere is always perpendicular to the vector $\overrightarrow{OP}$ from the center O of the sphere to the point P.

3. Consider the surface $z = f(x, y)$, and a given vector $(a, b) \in \mathbb{R}^2$. What is a tangent vector at $(x_0, y_0, f(x_0, y_0))$ that has (a, b) as its (x, y)-component?

4. Let $f(x, y) = x^2 + y^2$.

   (a) Describe the level curve $f(x, y) = 2$.

   (b) Without calculation, find the directional derivative of f at $(1, 1)$ in the direction $\frac{1}{\sqrt{2}}(-1, 1)$.

   (c) By computation, find the directional derivative at $(1, 1)$ in the direction of $\frac{1}{\sqrt{2}}(-1, 1)$

5. In the picture below, equispaced level curves of a function $f(x, y)$ are given.

   (a) Where is ∇f largest in magnitude?

   (b) Where is ∇f smallest in magnitude?

   (c) For what (x, y) is $f_x(x, y)$ equal to 0?

   (d) Where is the directional derivative $D_{(1/\sqrt{2},1/\sqrt{2})}f$ equal to 0?