Problem Set 8 - Math Tutorial Calculus II

1. Show that the two surfaces $z = xy$ and $z = \frac{3}{4}x^2 - y^2$ intersect perpendicularly at the point $(2, 1, 2)$.

2. Verify the sum rule for the derivative matrices for $f(x, y) = xy + \cos(x)$ and $g(x, y) = \sin(xy) + y^3$.

3. Verify the product and quotient rules for $f(x, y) = x^2y + y^3$ and $g(x, y) = \frac{x}{y}$.

4. Determine all second-order partial derivatives of $f(x, y) = \frac{1}{\sin^2 x + 2e^y}$.

5. If $f(x, y, z) = x^2 - y^3 + xyz$, and $x = 6t + 7$, $y = \sin(2t)$, $z = t^2$, verify the chain rule by finding $\frac{df}{dt}$ in two different ways.

6. Calculate $D(\vec{f} \circ \vec{g})$ in two ways: (a) by first evaluating $\vec{f} \circ \vec{g}$ and (b) by using chain rule and the derivative matrices $D\vec{f}$ and $D\vec{g}$:

(i) $\vec{f}(x) = (3x^5, e^{2x})$, $\vec{g}(s, t) = s - 7t$,

(ii) $\vec{f}(x, y, z) = (x + y + z, x^3 - e^{yz})$, $\vec{g}(s, t, u) = (st, tu, su)$.

7. Find $\frac{dy}{dx}$ when y is defined implicitly by the equation $\sin(xy) - x^2y^7 + e^y = 0$.

8. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, where z is given implicitly by the equation $x^3z + y\cos(z) + \frac{\sin(y)}{z} = 0$.