

Problem Set 4 - Math Tutorial Calculus II

1. Sketch the solid whose cylindrical coordinates (r, θ, z) satisfy

$$r \leq z \leq 5, \quad 0 \leq \theta \leq \pi.$$

2. Consider the following ice-cream-cone like solid (see sketch), and describe it using (a) spherical coordinates, and (b) cylindrical coordinates.

3. Consider the function $f(x, y) = 2 + \ln(x^2 + y^2)$.

(a) Sketch some level curves and sections of f .

(b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.

4. Consider the function $f(x, y) = \cos \sqrt{x^2 + y^2}$.

(a) Sketch some level curves and sections of f .

(b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.

5. Consider the function $f(x, y) = \frac{1}{x^2 + y^2 + 4}$.

(a) Sketch some level curves and sections of f .

(b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.

6. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = 2x^2 + 1$.

(a) Find the domain and range of f .

(b) Is f one-to-one?

(c) Is f onto?