Problem Set 13 - Math Tutorial Calculus II

1.(a) Let f be a continuous function of one variable. Show that if f has tow local maxima, then f must also have a local minimum.

(b) The analogue of part (a) does not necessarily hold for continuous functions of more than one variable. Consider the function

$$f(x,y) = 2 - (xy^2 - y - 1)^2 - (y-1)^2.$$

Show that f has just two critical points - and that both of them are local maxima.

(c) Draw the graph of a function of two variables which has two local maxima but not local minimum.

2. Show that the largest rectangular box having a fixed surface area must be a cube.

3. True or false?

(a) Every rectangle in $\mathbb{R}^2$ may be denoted by $[a,b] \times [c,d]$.

(b) $\int_0^2 \int_0^x 3\ dydx = \int_0^2 \int_0^y 3\ dxdy$.

(c) $\int_{-1}^1 \int_0^3 x^2 e^{x+y}\ dydx = \left(\int_{-1}^1 x^2 e^x\ dx\right)\left(\int_0^3 e^y\ dy\right)$

(d) The region in $\mathbb{R}^2$ bounded by the graphs of $y = x^3$ and $y = \sqrt{x}$ is a type 3 elementary region in the plane.

(e) The increment $\triangle f$ of a function $f(x,y)$ measure the change in the z-coordinate of the tangent plane of the graph of f.

(f) If $\det Hf(\vec{a}) = 0$, then f has a saddle point.

(g) Let f be a differentiable function of two variables. The slope of the line tangent to the curve obtained when intersecting the graph of f with the plane $x+y=0$ at the point $(a,-a,f(a,-a))$ equals the directional derivative of f at $(a,-a,f(a,-a))$ in the direction $\vec{i}+\vec{j}$.