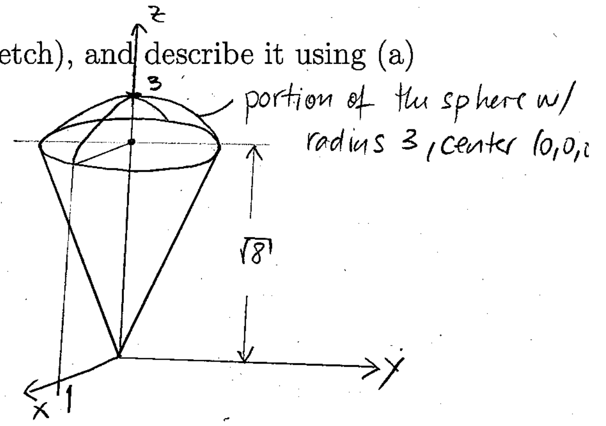
Problem Set 4 - Math Tutorial Calculus II

1. Sketch the solid whose cylindrical coordinates (r, θ, z) satisfy

$$r \leq z \leq 5, \quad 0 \leq \theta \leq \pi.$$

2. Consider the following ice-cream-cone like solid (see sketch), and describe it using (a) spherical coordinates, and (b) cylindrical coordinates.



3. Consider the function $f(x, y) = 2 + \ln(x^2 + y^2)$.

   (a) Sketch some level curves and sections of f.

   (b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.

4. Consider the function $f(x, y) = \cos\sqrt{x^2 + y^2}$.

   (a) Sketch some level curves and sections of f.

   (b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.

5. Consider the function $f(x, y) = \frac{1}{x^2+y^2+4}$.

   (a) Sketch some level curves and sections of f.

   (b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 2x^2 + 1$.

   (a) Find the domain and range of f.

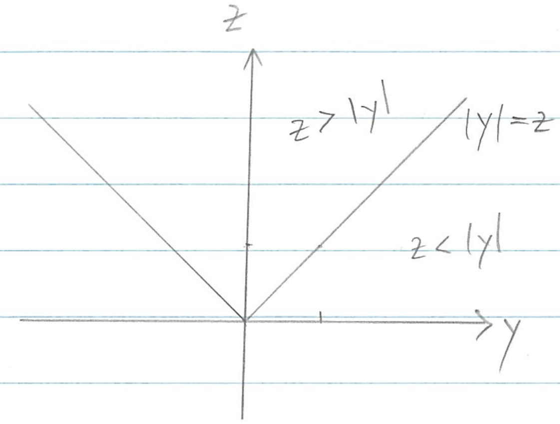
   (b) Is f one-to-one?

   (c) Is f onto?

Problem set 4 - Solutions

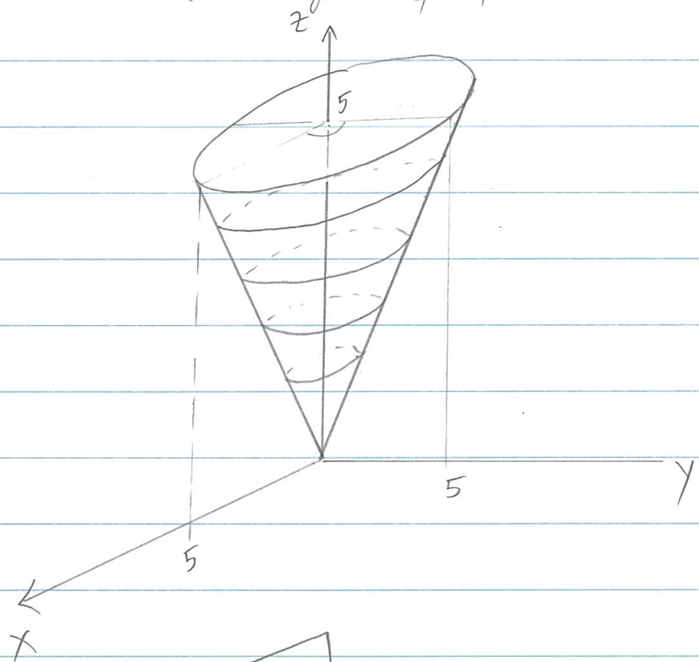
1) $r \leq z \leq 5$, $0 \leq \theta \leq \pi$

$r \leq z \Rightarrow (x^2+y^2)^{1/2} \leq z$

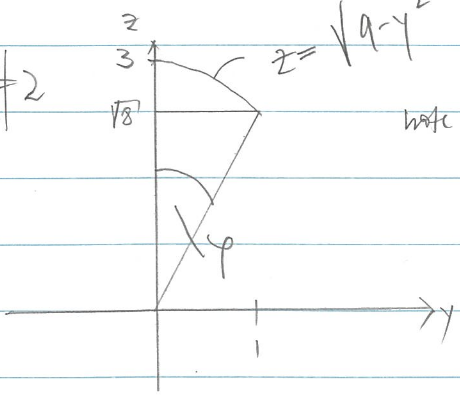


$\Rightarrow$ cone with central axis = z-axis,

vertex = origin, angle = $\frac{\pi}{4}$, bounded from above by $z=5$



#2



note: $1^2+(\sqrt{8})^2 = 9 = 3^2$

(a) $0 \leq \rho \leq 3$, $0 \leq \varphi \leq \tan^{-1}\left(\frac{1}{\sqrt{8}}\right)$

$0 \leq \theta \leq 2\pi$

(b) $0 \leq \theta < 2\pi$, $0 \leq r \leq 1$

$\sqrt{8}\, r \leq z \leq \sqrt{9-r^2}$

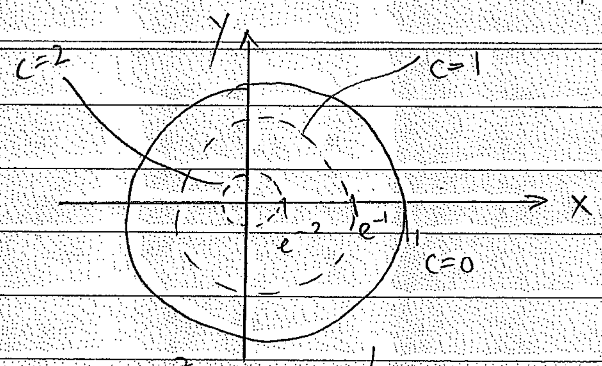
#3

$$f(x,y) = 2 + \ln(x^2+y^2)$$

(a) $c=1 \Rightarrow \ln(x^2+y^2) = -1 \Rightarrow x^2+y^2 = e^{-1}$

$c=0 \Rightarrow \ln(x^2+y^2) = -2 \Rightarrow x^2+y^2 = e^{-2}$

$c=2 \Rightarrow \ln(x^2+y^2) = 0 \Rightarrow x^2+y^2 = 1$



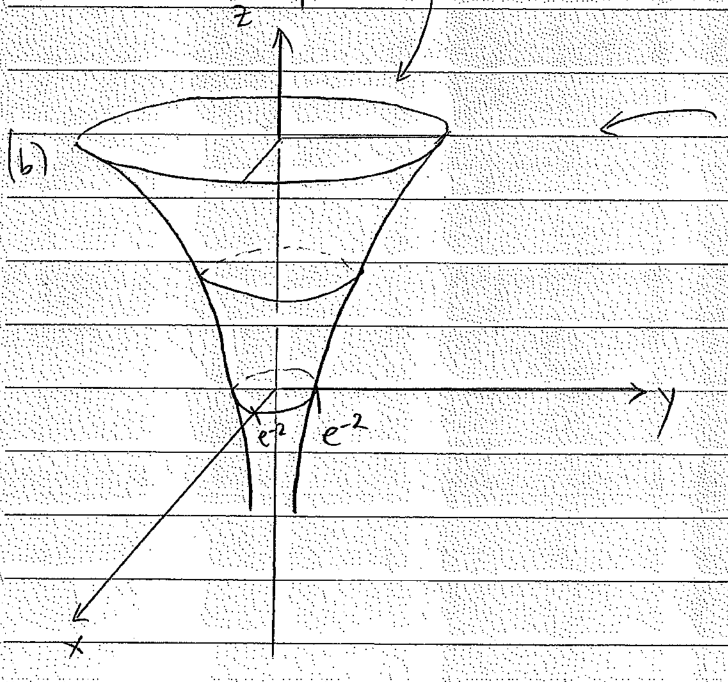
$y=0 \Rightarrow z = 2+\ln x^2 = 2\ln|x|+2$

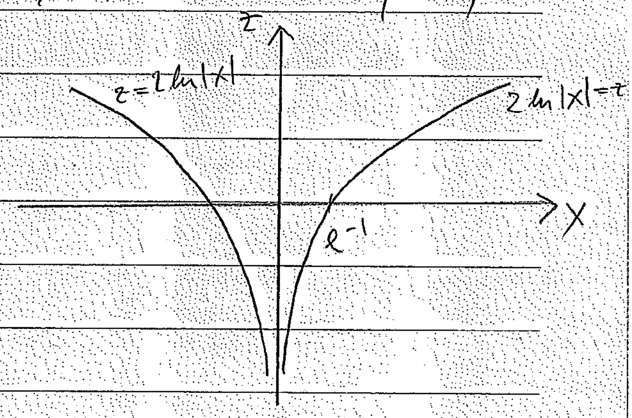
$y=1 \Rightarrow z = \ln(x^2+1)+2$

$x=0 \Rightarrow z = 2\ln|y|+2$

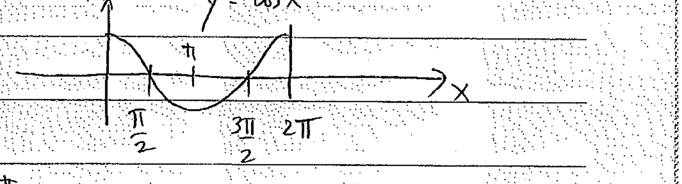
$x=1 \Rightarrow z = \ln(y^2+1)+2$

(b)





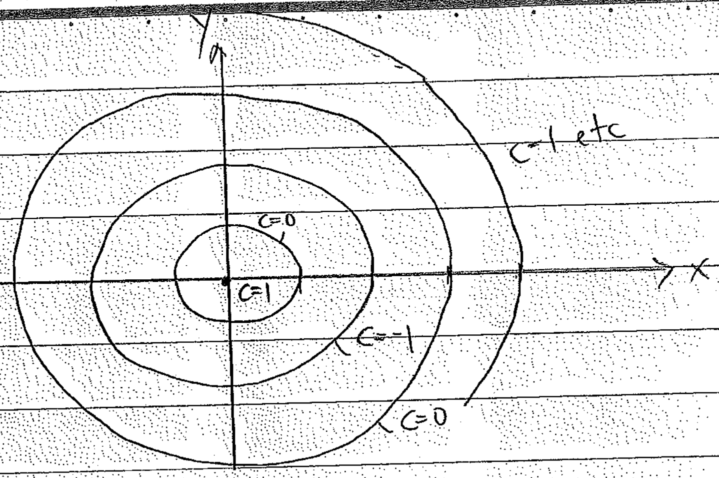
#4 $f(x,y) = \cos\left((x^2+y^2)^{1/2}\right)$



(a) $c=0 \Rightarrow (x^2+y^2)^{1/2} = \frac{\pi}{2} + n\pi \quad n\in\mathbb{Z}$

$c=1 \Rightarrow (x^2+y^2)^{1/2} = 2\pi n \quad n\in\mathbb{Z}$

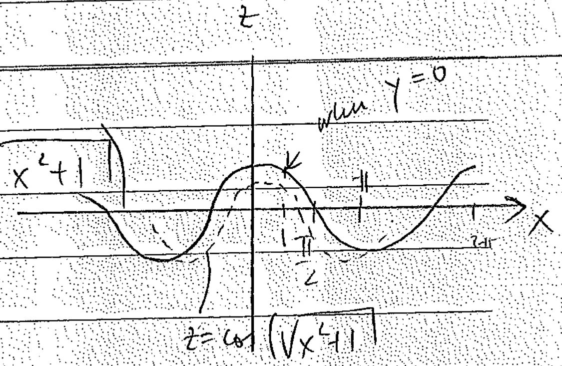
$c=-1 \Rightarrow (x^2+y^2)^{1/2} = \pi + 2\pi n \quad n\in\mathbb{Z}$



Some level sets

Some sections: $y=0 \Rightarrow z = \cos|x|$

$y=1 \Rightarrow z = \cos\left(\left|\sqrt{x^2+1}\right|\right)$



It looks similar to the surface of water after throwing a stone in,
see pg. figure B on pg. 172 (this is for sin instead of cos)

#5 $f(x,y) = \dfrac{1}{x^2+y^2+4}$

(a) level curves at height $c>0$ are circles: $c = f(x,y)$

$\Rightarrow x^2+y^2+4 = \frac{1}{c}$

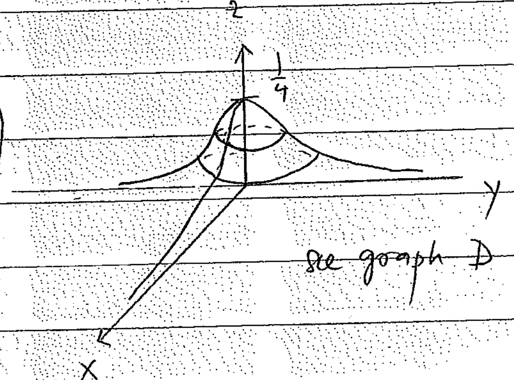
$\Rightarrow x^2+y^2 = \frac{1}{c} - 4 \Rightarrow$

necessarily $\frac{1}{c} - 4 \geq 0$

$\Rightarrow \frac{1}{c} \geq 4$

$\Rightarrow c \leq \frac{1}{4}$

(b)



see graph D on pg. 172

#6 $f: \mathbb{R} \to \mathbb{R}$ with $f(x) = 2x^2+1$

$\Rightarrow$ (a) $\mathrm{Dom}(f) = \mathbb{R}$, $\mathcal{R}(f) = \{ y \in \mathbb{R} : y \geq 1 \}$

because $\sqrt{(y-1)\cdot\frac{1}{2}} \in \mathbb{R}$ for $y \geq 1$.

(b) f is not 1-1 since $f(1) = 3 = f(-1)$

(c) f is not onto since for instance $0 \in \mathbb{R}$ (= codomain)

but $0 \notin \mathcal{R}(f)$.