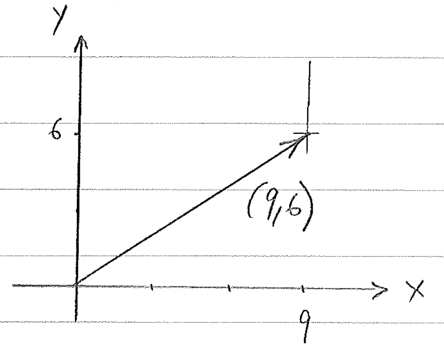
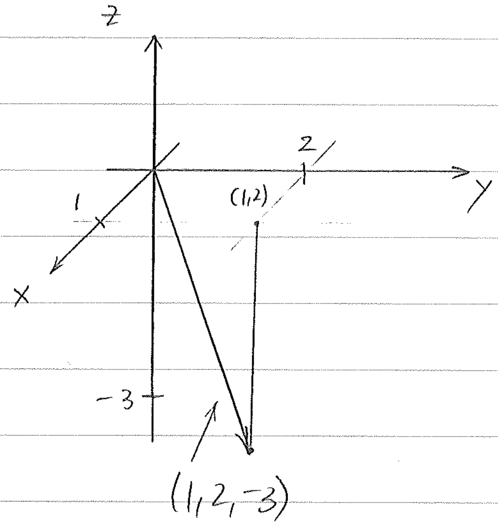
# Problem set 1 - Solutions

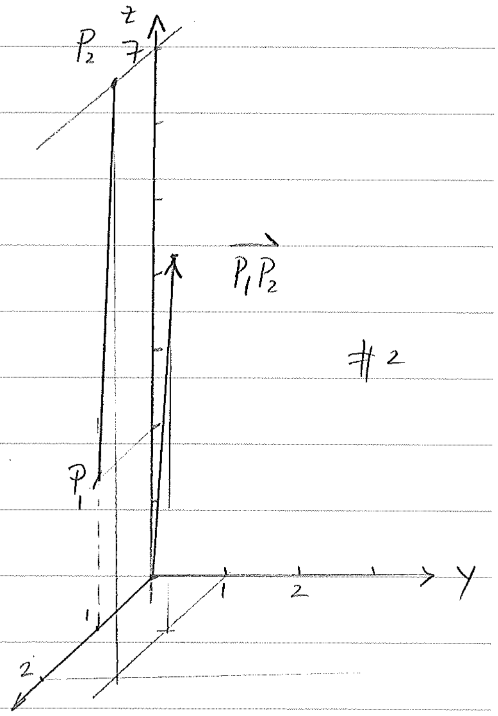
#1 $(9,6) = 9\vec{i} + 6\vec{j}$ , $(1,2,-3) = \vec{i} + 2\vec{j} - 3\vec{k}$



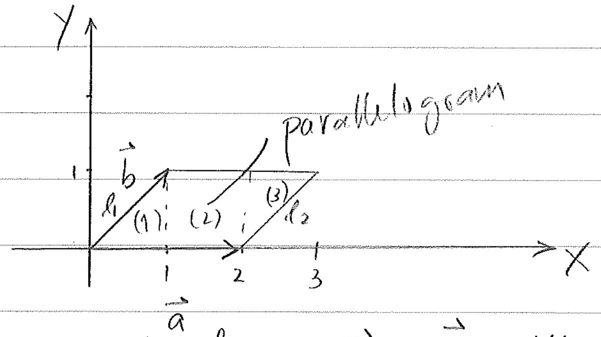

#2 $P_1(1,0,2)$ , $P_2(2,1,7)$

$\Rightarrow$ displacement vector $\overrightarrow{P_1P_2} = (1,1,5)$




#3 $\vec{a} = (2,0)$, $\vec{b} = (1,1)$, $0 \le s \le 1$, $0 \le t \le 1$

Let $\vec{x} = s\vec{a} + t\vec{b}$




we may identify $s\vec{a} + t\vec{b}$ with the point $(2s+t, t)$ and note that

$l_1: \ y = x$ , $l_2: \ y = x-2 \Leftrightarrow x = y+2$

$\Rightarrow (x,y)$ is in the parallelogram if $0 \le y \le 1$ and $y \le x \le y+2$

$(2s+t, t) = (x,y) \Rightarrow 0 \le y \le 1$ and $t \le 2s+t \le t+2$ since $0 \le s \le 1$

#4 Line through $(12, -2, 0)$, parallel to $5\vec{i} - 12\vec{j} + \vec{k}$

$$\Rightarrow \begin{cases} x = 12 + 5t \\ y = -2 - 12t \\ z = t \end{cases}$$ parametric equations;

symmetric equations: $\dfrac{x-12}{5} = \dfrac{y+2}{-12} = z$.

#5 line $\begin{cases} x = 3t - 5 \\ y = 2 - t \\ z = 6t \end{cases}$, plane $x + 3y - z = 19$.

$P = (x(t_0), y(t_0), z(t_0))$ is a point on the plane if $x(t_0) + 3y(t_0) - z(t_0) = 19$.

So compute: $(3t-5) + 3(2-t) - (6t)$
$= 3t - 3t - 6t - 5 + 6 = -6t + 1$

$-6t + 1 = 19$ if $t = -3 \Rightarrow$ The line and the plane intersect at $(-14, 5, -18)$. (How do you check this?)

#6 The equations $x = 5t^2 - 1$, $y = t^2 + 3$ and $z = 1 - t^2$ determine a line since

$$t^2 = \underbrace{\frac{x+1}{5} = y - 3 = -z + 1}_{\text{these are symmetric eqns for a line.}}$$

#7 line $\begin{cases} x = 2t-3 \\ y = 3t+2 \\ z = 5-t \end{cases}$ , planes : $x=0$, $y=0$, $z=0$

· Intersection with the plane $x=0$ : $0 = 2t-3 \Rightarrow t = \frac{3}{2}$

$$\Rightarrow y = \frac{9}{2} + 2 = \frac{13}{2}$$

$$z = 5 - \frac{3}{2} = \frac{7}{2}$$

$\Rightarrow$ pt. of intersection $\left(0, \frac{13}{2}, \frac{7}{2}\right)$

· intersection with the plane $y=0$ : $0 = 3t+2 \Rightarrow t = -\frac{2}{3}$

$$\Rightarrow x = -\frac{4}{3} - 3 = -\frac{13}{3}$$

$$z = 5 + \frac{2}{3} = \frac{17}{3}$$

$\Rightarrow$ pt. of intersection $\left(-\frac{13}{3}, 0, \frac{17}{3}\right)$

· intersection with the plane $z=0$ : $0 = 5-t \Rightarrow t=5$

$$\Rightarrow x = 10-3 = 7$$

$$y = 15+2 = 17$$

$\Rightarrow$ pt. of intersection $(7, 17, 0)$.