hw9

[2.6: 32, 34]

32. Since $x^2 - x^3 = y^2$, putting $f(x,y) = x^2 - x^3 - y^2$, the curve can be expressed as $f(x,y) = 0$.

$$\nabla f(-1, \sqrt{2}) = (2x - 3x^2, -2y)\Big|_{(x,y)=(-1,\sqrt{2})} = (-2-3, -2\sqrt{2}) = (-5, -2\sqrt{2})$$

Thus, the normal line to the curve $x^2 - x^3 = y^2$ at $(-1, \sqrt{2})$ can be expressed as:

$$\begin{cases} x = -5t - 1 \\ y = -2\sqrt{2}t + \sqrt{2} \end{cases} \quad \text{(parametric equations)}$$

or

$$2\sqrt{2}x = -10\sqrt{2}t - 2\sqrt{2}$$

$$\underline{5y = -10\sqrt{2}t + 5\sqrt{2}} \; -$$

$$2\sqrt{2}x - 5y = -7\sqrt{2}$$

$$2\sqrt{2}x - 5y + 7\sqrt{2} = 0 \qquad \text{(Cartesian equation)}$$

34. $S: x^3 z + x^2 y^2 + \sin yz = -3$.

(a) Put $f(x,y,z) = x^3 z + x^2 y^2 + \sin yz$

$$\nabla f(-1,0,3) = (3x^2 z + 2xy^2,\ 2x^2 y + z\cos yz,\ x^3 + y\cos yz)\Big|_{(x,y,z)=(-1,0,3)}$$

$$= (9 + 0,\ 0 + 3\cdot 1,\ -1 + 0) = (9, 3, -1).$$

$$\nabla f(-1,0,3) \cdot (x+1, y, z-3) = 0$$

$$(9,3,-1) \cdot (x+1, y, z-3) = 0$$

$$9x + 9 + 3y - z + 3 = 0$$

$9x + 3y - z + 12 = 0$ gives an equation for the plane tangent to S at $(-1, 0, 3)$

(b) $\nabla f(-1,0,3) = (9,3,-1)$ gives a vector normal to S at $(-1,0,3)$.

Since the line normal to S at $(-1,0,3)$ passes $(-1,0,3)$, the normal line to S at $(-1,0,3)$ can be written as:

$$\begin{cases} x = 9t - 1 \\ y = 3t \\ z = -t + 3 \end{cases}$$

[4.1: 2, 6, 8, 10, 14, 16, 22]

2. $f(x) = \ln(1+x)$, $a=0$, $k=3$.

$P_3(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2 + \frac{1}{3!} f'''(0)x^3$

$f(0) = \ln 1 = 0$,

$f'(0) = \frac{1}{1+x}\Big|_{x=0} = 1$,

$f''(0) = -\frac{1}{(1+x)^2}\Big|_{x=0} = -1$,

$f'''(0) = \frac{2}{(1+x)^3}\Big|_{x=0} = 2$

Hence, $P_3(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3$ //

6. $f(x) = \sin x$, $a=0$, $k=5$.

$P_5(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2 + \frac{1}{3!} f'''(0)x^3 + \frac{1}{4!} f^{(4)}(0)x^4 + \frac{1}{5!} f^{(5)}(0)x^5$.

$f(0) = \sin 0 = 0$

$f'(0) = \cos x|_{x=0} = 1$, $f''(0) = -\sin x|_{x=0} = 0$, $f'''(0) = -\cos x|_{x=0} = -1$

$f^{(4)}(0) = \sin x|_{x=0} = 0$, $f^{(5)}(0) = \cos x|_{x=0} = 1$.

Thus $P_5(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$ //

8. $f(x,y) = \frac{1}{x^2+y^2+1}$, $\vec{a} = (0,0)$

$P_1(0,0) = f(0,0) + Df(0,0)\begin{pmatrix} x \\ y \end{pmatrix}$, $P_2(0,0) = f(0,0) + Df(0,0)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x\ y)Hf(0,0)\begin{pmatrix} x \\ y \end{pmatrix}$

$f(0,0) = 1$

$Df(0,0) = \left(-\frac{2x}{(x^2+y^2+1)^2} \quad -\frac{2y}{(x^2+y^2+1)^2}\right)\Big|_{(x,y)=(0,0)} = (0\ 0)$

$f_{xx} = \frac{\partial}{\partial x}\left(-\frac{2x}{(x^2+y^2+1)^2}\right) = -2(x^2+y^2+1)^{-2} - 2x(-2)(2x)(x^2+y^2+1)^{-3}$

$= -\frac{2}{(x^2+y^2+1)^2} + \frac{8x^2}{(x^2+y^2+1)^3} = \frac{-2(x^2+y^2+1)+8x^2}{(x^2+y^2+1)^3}$

$= \frac{6x^2-2y^2-2}{(x^2+y^2+1)^3}$

$f_{yy} = \frac{\partial}{\partial y}\left(-\frac{2y}{(x^2+y^2+1)^2}\right) = \frac{-2x^2+6y^2-2}{(x^2+y^2+1)^3}$

$$f_{xy} = f_{yx} = 8xy\,(x^2+y^2+1)^{-3} = \frac{8xy}{(x^2+y^2+1)^3}$$

$$Hf(0,0) = \begin{pmatrix} f_{xx}(0,0) & f_{xy}(0,0) \\ f_{yx}(0,0) & f_{yy}(0,0) \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\therefore\ P_1(x,y) = 1 /\!/$$

$$P_2(x,y) = 1 + \frac{1}{2}(x\ \ y)\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1 + \frac{1}{2}(-2x\ \ -2y)\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= 1 + \frac{1}{2}(-2x^2 - 2y^2) = 1 - x^2 - y^2 /\!/$$

10. $f(x,y) = e^{2x+y}$, $\vec{a} = (0,0)$

$$f_x = 2e^{2x+y}, \quad f_{xx} = 4e^{2x+y}, \quad f_{xy} = f_{yx} = 2e^{2x+y}$$

$$f_y = e^{2x+y}, \quad f_{yy} = e^{2x+y}$$

$$f(0,0) = e^0 = 1$$

$$Df(0,0) = (f_x(0,0)\ \ f_y(0,0)) = (2\ \ 1)$$

$$Hf(0,0) = \begin{pmatrix} f_{xx}(0,0) & f_{xy}(0,0) \\ f_{yx}(0,0) & f_{yy}(0,0) \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$$

$$\therefore\ P_1(x,y) = f(0,0) + Df(0,0)\begin{pmatrix} x \\ y \end{pmatrix} = 1 + (2\ 1)\begin{pmatrix} x \\ y \end{pmatrix} = 1 + 2x + y /\!/$$

$$P_2(x,y) = f(0,0) + Df(0,0)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x\ \ y)\,Hf(0,0)\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= 1 + (2\ 1)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x\ y)\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= 1 + 2x + y + \frac{1}{2}(4x+2y\ \ \ 2x+y)\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= 1 + 2x + y + \frac{1}{2}(4x^2 + 2xy + 2xy + y^2)$$

$$= 1 + 2x + y + \frac{1}{2}(4x^2 + 4xy + y^2) = 1 + 2x + y + \frac{1}{2}(2x+y)^2 /\!/$$

14. $f(x,y) = \dfrac{1}{x^2+y^2+1}$, $\vec{a} = (0,0)$

$$f_x = \frac{-2x}{(x^2+y^2+1)^2}, \quad f_y = \frac{-2y}{(x^2+y^2+1)^2}$$

$$f_{xx} = \frac{6x^2 - 2y^2 - 2}{(x^2+y^2+1)^3}, \quad f_{xy} = f_{yx} = \frac{8xy}{(x^2+y^2+1)^3}, \quad f_{yy} = \frac{-2x^2+6y^2-2}{(x^2+y^2+1)^3}$$

$$Hf(0,0) = \begin{pmatrix} f_{xx}(0,0) & f_{xy}(0,0) \\ f_{yx}(0,0) & f_{yy}(0,0) \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$ //

16. $f(x,y,z) = e^{2x-3y}\sin 5z$, $\vec{a} = (0,0,0)$

$f_x = 2e^{2x-3y}\sin 5z$, $f_{xx} = 4e^{2x-3y}\sin 5z$, $f_{xy} = f_{yx} = -6e^{2x-3y}\sin 5z$

$f_y = -3e^{2x-3y}\sin 5z$, $f_{yy} = 9e^{2x-3y}\sin 5z$, $f_{yz} = f_{zy} = -15e^{2x-3y}\cos 5z$

$f_z = 5e^{2x-3y}\cos 5z$, $f_{zz} = -25e^{2x-3y}\sin 5z$, $f_{xz} = f_{zx} = 10e^{2x-3y}\cos 5z$

$$Hf(0,0,0) = \begin{pmatrix} f_{xx}(0,0,0) & f_{xy}(0,0,0) & f_{xz}(0,0,0) \\ f_{yx}(0,0,0) & f_{yy}(0,0,0) & f_{yz}(0,0,0) \\ f_{zx}(0,0,0) & f_{zy}(0,0,0) & f_{zz}(0,0,0) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 10 \\ 0 & 0 & -15 \\ 10 & -15 & 0 \end{pmatrix}$$ //

22. $f(x,y) = x^2y^3$

$df(x,y) = f_x\,dx + f_y\,dy = 2xy^3\,dx + 3x^2y^2\,dy$ //