HW8

[25: 16, 18, 20, 25]

16. $f(x,y) = x^2 - 3y^2, \quad \vec{g}(s,t) = (st,\ s+t^2)$

(a) $$f\circ\vec{g}\,(s,t) = f(st,\ s+t^2) = (st)^2 - 3(s+t^2)^2 = s^2t^2 - 3(s^2+2st^2+t^4)$$
$$= s^2t^2 - 3s^2 - 6st^2 - 3t^4$$

$$D(f\circ\vec{g})(s,t) = (2st^2 - 6s - 6t^2,\quad 2s^2t - 12st - 12t^3) \ //$$

(b) $$D(f\circ\vec{g})(s,t) = Df(x,y)\,D\vec{g}(s,t)$$
$$= \begin{pmatrix} 2x & -6y \end{pmatrix} \begin{pmatrix} t & s \\ 1 & 2t \end{pmatrix} = (2xt - 6y,\ 2xs - 12yt)$$
$$= (2(st)t - 6(s+t^2),\quad 2(st)s - 12(s+t^2)t)$$
$$= (2st^2 - 6s - 6t^2,\quad 2s^2t - 12st - 12t^3) \ //$$

18. $\vec{f}(x,y,z) = (x^2y + y^2z,\ xyz,\ e^z)$

$\vec{g}(t) = (t-2,\ 3t+7,\ t^3)$

(a) $$\vec{f}\circ\vec{g}\,(t) = \vec{f}(t-2,\ 3t+7,\ t^3) = \left((t-2)^2(3t+7) + (3t+7)^2t^3,\ (t-2)(3t+7)t^3,\ e^{t^3}\right)$$
$$= \left((t^2-4t+4)(3t+7) + (3t+7)(3t^4+7t^3),\ (3t^2-6t+7t-14)t^3,\ e^{t^3}\right)$$
$$= \left((3t+7)(t^2-4t+4+3t^4+7t^3),\ t^3(3t^2+t-14),\ e^{t^3}\right)$$
$$= \left((3t+7)(3t^4+7t^3+t^2-4t+4),\ t^3(3t^2+t-14),\ e^{t^3}\right)$$

$$D(\vec{f}\circ\vec{g})(t) = \begin{pmatrix} 3(3t^4+7t^3+t^2-4t+4) + (3t+7)(12t^3+21t^2+2t-4) \\ 3t^2(3t^2+t-14) + t^3(6t+1) \\ 3t^2e^{t^3} \end{pmatrix} //$$

(b) $$D(\vec{f}\circ\vec{g})(t) = D\vec{f}(x,y,z)\,D\vec{g}(t) = \begin{pmatrix} 2xy & x^2+2yz & y^2 \\ yz & xz & xy \\ 0 & 0 & e^z \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 3t^2 \end{pmatrix}$$

$$= \begin{pmatrix} 2xy + 3(x^2+2yz) + 3y^2t^2 \\ yz + 3xz + 3xyt^2 \\ 3t^2e^z \end{pmatrix}$$

$$= \begin{pmatrix} 2(t-2)(3t+7) + 3\left((t-2)^2 + 2(3t+7)t^3\right) + 3(3t+7)^2t^2 \\ (3t+7)t^3 + 3(t-2)t^3 + 3(t-2)(3t+7)t^2 \\ 3t^2e^{t^3} \end{pmatrix} //$$

20. $\vec{g}: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ $\qquad$ $\vec{f}: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$\vec{g}(1,-1,3) = (2,5)$ $\qquad$ $\vec{f}(x,y) = (2xy,\ 3x-y+5)$

$$D\vec{g}(1,-1,3) = \begin{pmatrix} 1 & -1 & 0 \\ 4 & 0 & 7 \end{pmatrix}$$

$$D(\vec{f}\circ\vec{g})(1,-1,3) = D\vec{f}(x,y)\, D\vec{g}(s,t,u)\Big|_{(s,t,u)=(1,-1,3)}$$

$$= \begin{pmatrix} 2y & 2x \\ 3 & -1 \end{pmatrix}\Bigg|_{(x,y)=(2,5)} D\vec{g}(1,-1,3)$$

$$= \begin{pmatrix} 10 & 4 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 0 \\ 4 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 10+16 & -10 & 28 \\ 3-4 & -3 & -7 \end{pmatrix}$$

$$= \begin{pmatrix} 26 & -10 & 28 \\ -1 & -3 & -7 \end{pmatrix} //$$

25. $\begin{Bmatrix} \rho^2 = r^2+z^2 \\ \tan\varphi = r/z \\ \theta = \theta \end{Bmatrix} \Longrightarrow \begin{Bmatrix} 2\rho\frac{\partial\rho}{\partial r} = 2r, & 2\rho\frac{\partial\rho}{\partial z} = 2z \\ \sec^2\varphi\frac{\partial\varphi}{\partial r} = \frac{1}{z}, & \sec^2\varphi\frac{\partial\varphi}{\partial z} = -\frac{r}{z^2} \\ \theta = \theta \end{Bmatrix}$ --- ①

$\begin{Bmatrix} r = \rho\sin\varphi \\ \theta = \theta \\ z = \rho\cos\varphi \end{Bmatrix}$ ... ② $\qquad \begin{Bmatrix} \frac{\partial}{\partial x} = \cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta} \\ \frac{\partial}{\partial y} = \sin\theta\frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta} \end{Bmatrix}$ --- ③

(a) $\frac{\partial}{\partial r} = \frac{\partial\rho}{\partial r}\frac{\partial}{\partial\rho} + \frac{\partial\varphi}{\partial r}\frac{\partial}{\partial\varphi} \overset{①}{=} \frac{r}{\rho}\frac{\partial}{\partial\rho} + \frac{\cos^2\varphi}{z}\frac{\partial}{\partial\varphi} \overset{②}{=} \sin\varphi\frac{\partial}{\partial\rho} + \frac{\cos\varphi}{\rho}\frac{\partial}{\partial\varphi}$ //

(b) $\frac{\partial}{\partial z} = \frac{\partial\rho}{\partial z}\frac{\partial}{\partial\rho} + \frac{\partial\varphi}{\partial z}\frac{\partial}{\partial\varphi} \overset{①}{=} \frac{z}{\rho}\frac{\partial}{\partial\rho} - \frac{r\cos^2\varphi}{z^2}\frac{\partial}{\partial\varphi} \overset{②}{=} \cos\varphi\frac{\partial}{\partial\rho} - \frac{\rho\sin\varphi\cos^2\varphi}{\rho^2\cos^2\varphi}\frac{\partial}{\partial\varphi}$

$$= \cos\varphi\frac{\partial}{\partial\rho} - \frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi} \quad ---\ ③'$$

Next we use ③, ③', and the result of (a) to compute $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

③,③'

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} = \left(\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\left(\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)$$

$$+\left(\sin\theta\frac{\partial}{\partial r}+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right)\left(\sin\theta\frac{\partial}{\partial r}+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right)$$

$$+\left(\cos\varphi\frac{\partial}{\partial\rho}-\frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}\right)\left(\cos\varphi\frac{\partial}{\partial\rho}-\frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}\right)$$

$$=\cos\theta\frac{\partial}{\partial r}\left(\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)$$

$$+\sin\theta\frac{\partial}{\partial r}\left(\sin\theta\frac{\partial}{\partial r}+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right)+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial r}+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right)$$

$$+\cos\varphi\frac{\partial}{\partial\rho}\left(\cos\varphi\frac{\partial}{\partial\rho}-\frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}\right)-\frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}\left(\cos\varphi\frac{\partial}{\partial\rho}-\frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}\right)$$

$$=\cos^2\theta\frac{\partial^2}{\partial r^2}+\cancel{\frac{\cos\theta\sin\theta}{r^2}}\frac{\partial}{\partial\theta}-\cancel{\frac{\cos\theta\sin\theta}{r}}\frac{\partial^2}{\partial r\partial\theta}$$

$$+\frac{\sin^2\theta}{r}\frac{\partial}{\partial r}-\cancel{\frac{\sin\theta\cos\theta}{r}}\frac{\partial^2}{\partial\theta\partial r}+\cancel{\frac{\sin\theta\cos\theta}{r^2}}\frac{\partial}{\partial\theta}+\frac{\sin^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2}$$

$$+\sin^2\theta\frac{\partial^2}{\partial r^2}-\cancel{\frac{\sin\theta\cos\theta}{r^2}}\frac{\partial}{\partial\theta}+\cancel{\frac{\sin\theta\cos\theta}{r}}\frac{\partial^2}{\partial r\partial\theta}$$

$$+\frac{\cos^2\theta}{r}\frac{\partial}{\partial r}+\cancel{\frac{\cos\theta\sin\theta}{r}}\frac{\partial^2}{\partial\theta\partial r}-\cancel{\frac{\cos\theta\sin\theta}{r^2}}\frac{\partial}{\partial\theta}+\frac{\cos^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2}$$

$$+\cos^2\varphi\frac{\partial^2}{\partial\rho^2}+\frac{\cos\varphi\sin\varphi}{\rho^2}\frac{\partial}{\partial\varphi}-\frac{\cos\varphi\sin\varphi}{\rho}\frac{\partial^2}{\partial\rho\partial\varphi}$$

$$+\frac{\sin^2\varphi}{\rho}\frac{\partial}{\partial\rho}-\frac{\sin\varphi\cos\varphi}{\rho}\frac{\partial^2}{\partial\varphi\partial\rho}+\frac{\sin\varphi\cos\varphi}{\rho^2}\frac{\partial}{\partial\varphi}+\frac{\sin^2\varphi}{\rho^2}\frac{\partial^2}{\partial\varphi^2}$$

$$=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\cos^2\varphi\frac{\partial^2}{\partial\rho^2}+\frac{\sin^2\varphi}{\rho}\frac{\partial}{\partial\rho}+\frac{2\cos\varphi\sin\varphi}{\rho^2}\frac{\partial}{\partial\varphi}$$

$$-\frac{\cos\varphi\sin\varphi}{\rho}\frac{\partial^2}{\partial\rho\partial\varphi}-\frac{\sin\varphi}{\rho}\frac{\partial^2}{\partial\varphi\partial\rho}+\frac{\sin^2\varphi}{\rho^2}\frac{\partial^2}{\partial\varphi^2}$$

(a),②

$$=\left(\sin\varphi\frac{\partial}{\partial\rho}+\frac{\cos\varphi}{\rho}\frac{\partial}{\partial\varphi}\right)\left(\sin\varphi\frac{\partial}{\partial\rho}+\frac{\cos\varphi}{\rho}\frac{\partial}{\partial\varphi}\right)+\frac{1}{\rho\sin\varphi}\left(\sin\varphi\frac{\partial}{\partial\rho}+\frac{\cos\varphi}{\rho}\frac{\partial}{\partial\varphi}\right)$$

$$+\frac{1}{\rho^2\sin^2\varphi}\frac{\partial^2}{\partial\theta^2}+\cos^2\varphi\frac{\partial^2}{\partial\rho^2}+\frac{\sin^2\varphi}{\rho}\frac{\partial}{\partial\rho}+\frac{2\cos\varphi\sin\varphi}{\rho^2}\frac{\partial}{\partial\varphi}$$

$$-\frac{\cos\varphi\sin\varphi}{\rho}\frac{\partial^2}{\partial\rho\partial\varphi}-\frac{\sin\varphi\cos\varphi}{\rho}\frac{\partial^2}{\partial\varphi\partial\rho}+\frac{\sin^2\varphi}{\rho^2}\frac{\partial^2}{\partial\varphi^2}$$

$$= \sin^2\varphi \frac{\partial^2}{\partial\rho^2} - \cancel{\frac{\sin\varphi\cos\varphi}{\rho^2}} \frac{\partial}{\partial\varphi} + \cancel{\frac{\sin\varphi\cos\varphi}{\rho}} \frac{\partial^2}{\partial\rho\partial\varphi}$$

$$+ \frac{\cos^2\varphi}{\rho}\frac{\partial}{\partial\rho} + \cancel{\frac{\cos\varphi\sin\varphi}{\rho}}\frac{\partial^2}{\partial\varphi\partial\rho} - \cancel{\frac{\cos\varphi\sin\varphi}{\rho^2}}\frac{\partial}{\partial\varphi} + \frac{\cos^2\varphi}{\rho^2}\frac{\partial^2}{\partial\varphi^2}$$

$$+ \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{\cot\varphi}{\rho^2}\frac{\partial}{\partial\varphi} + \frac{1}{\rho^2\sin^2\varphi}\frac{\partial^2}{\partial\theta^2}$$

$$+ \cos^2\varphi\frac{\partial^2}{\partial\rho^2} + \frac{\sin^2\varphi}{\rho}\frac{\partial}{\partial\rho} + \cancel{\frac{2\cos\varphi\sin\varphi}{\rho^2}}\frac{\partial}{\partial\varphi}$$

$$- \cancel{\frac{\cos\varphi\sin\varphi}{\rho}}\frac{\partial^2}{\partial\rho\partial\varphi} - \cancel{\frac{\sin\varphi\cos\varphi}{\rho}}\frac{\partial^2}{\partial\varphi\partial\rho} + \frac{\sin^2\varphi}{\rho^2}\frac{\partial^2}{\partial\varphi^2}$$

$$= \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{\cot\varphi}{\rho^2}\frac{\partial}{\partial\varphi} + \frac{1}{\rho^2\sin^2\varphi}\frac{\partial^2}{\partial\theta^2}$$

$$= \frac{\partial^2}{\partial\rho^2} + \frac{2}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\cot\varphi}{\rho^2}\frac{\partial}{\partial\varphi} + \frac{1}{\rho^2\sin^2\varphi}\frac{\partial^2}{\partial\theta^2}$$

[2.6 : 2, 4, 6, 12, 14, 16, 20, 26]

2. $f(x,y) = e^y \sin x$, $\vec{a} = \left(\frac{\pi}{3}, 0\right)$, $\vec{u} = \frac{3\hat{i} - \hat{j}}{\sqrt{10}}$

$$D_{\vec{u}} f(\vec{a}) = \lim_{h\to 0} \frac{f(\vec{a}+h\vec{u}) - f(\vec{a})}{h} = \lim_{h\to 0} \frac{f\left(\left(\frac{\pi}{3},0\right) + \frac{h}{\sqrt{10}}(3,-1)\right) - f\left(\frac{\pi}{3},0\right)}{h}$$

$$= \lim_{h\to 0} \frac{f\left(\frac{\pi}{3} + \frac{3}{\sqrt{10}}h, -\frac{h}{\sqrt{10}}\right) - e^0 \sin\frac{\pi}{3}}{h}$$

$$= \lim_{h\to 0} \frac{e^{-h/\sqrt{10}} \sin\left(\frac{\pi}{3} + \frac{3h}{\sqrt{10}}\right) - \frac{1}{2}\sqrt{3}}{h}$$

$$= \lim_{h\to 0} \frac{-\frac{1}{\sqrt{10}} e^{-h/\sqrt{10}} \sin\left(\frac{\pi}{3} + \frac{3h}{\sqrt{10}}\right) + e^{-h/\sqrt{10}} \cdot \frac{3}{\sqrt{10}} \cos\left(\frac{\pi}{3} + \frac{3h}{\sqrt{10}}\right)}{1}$$

$$= -\frac{1}{\sqrt{10}} \sin\frac{\pi}{3} + \frac{3}{\sqrt{10}} \cos\frac{\pi}{3} = -\frac{1}{\sqrt{10}} \frac{1}{2}\sqrt{3} + \frac{3}{\sqrt{10}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{10}}\left(\sqrt{3} - 1\right) //$$

4. $f(x,y) = \frac{1}{x^2+y^2}$, $\vec{a} = (3,-2)$, $\vec{u} = \hat{i} - \hat{j}$

$$D_{\vec{u}} f(\vec{a}) = \lim_{h\to 0} \frac{f\left((3,-2) + \frac{h}{\sqrt{2}}(1,-1)\right) - f(3,-2)}{h} = \lim_{h\to 0} \frac{f\left(3+\frac{h}{\sqrt{2}}, -2-\frac{h}{\sqrt{2}}\right) - \frac{1}{13}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\left(3+\frac{h}{\sqrt{2}}\right)^2 + \left(2+\frac{h}{\sqrt{2}}\right)^2} - \frac{1}{13}}{h} = \lim_{h\to 0} \frac{\frac{1}{h^2+5\sqrt{2}h+13} - \frac{1}{13}}{h} = \lim_{h\to 0} \frac{-h^2-5\sqrt{2}h}{13h(h^2+5\sqrt{2}h+13)}$$

$$= \lim_{h\to 0} \frac{h(-h-5\sqrt{2})}{13h(h^2+5\sqrt{2}h+13)} = \lim_{h\to 0} \frac{-h-5\sqrt{2}}{13(h^2+5\sqrt{2}h+13)} = \frac{-5\sqrt{2}}{13\cdot 13} = -\frac{5\sqrt{2}}{169} //$$

6. $f(x,y,z) = xyz$, $\vec{a} = (-1,0,2)$, $\vec{u} = \frac{2\hat{k} - \hat{i}}{\sqrt{5}}$

$$D_{\vec{u}} f(\vec{a}) = \lim_{h\to 0} \frac{f\left((-1,0,2) + \frac{h}{\sqrt{5}}(-1,0,2)\right) - f(-1,0,2)}{h}$$

$$= \lim_{h\to 0} \frac{f\left(-1-\frac{h}{\sqrt{5}}, 0, 2+\frac{2h}{\sqrt{5}}\right) - 0}{h} = 0 //$$

12. $D_{\hat{\imath}} T(3,7) = 3$ deg/cm.

$D_{\hat{\jmath}} T(3,7) = -2$ deg/cm.

$$\begin{cases} D_{\hat{\imath}} T(3,7) = (T_x(3,7) \ \ T_y(3,7)) \cdot (1,0) = T_x(3,7) \\ D_{\hat{\jmath}} T(3,7) = (T_x(3,7) \ \ T_y(3,7)) \cdot (0,1) = T_y(3,7). \end{cases} \Rightarrow \nabla T(3,7) = (3,-2)$$

(a) The ladybug should move in the direction $\nabla T(3,7) = (3,-2)$ //

(b) The ladybug should move in the opposite direction of $\nabla T(3,7)$, that is $(-3,2)$ //

(c) The ladybug should move in the direction that is perpendicular to $\nabla T(3,7)$, that is, either $(2,3)$ or $(-2,-3)$. //

14. $R: \ 4x^2 + y^2 + 4z^2 = 16, \ z \geq 0.$

The surface is given by $z = \frac{1}{2}\sqrt{16-4x^2-y^2}$

Now $z(x,y)$ decreases most rapidly in the direction $-\nabla z(x,y) = (-z_x(x,y), -z_y(x,y))$.

Thus in the xy-plane the raindrop follows the curve with vector tangent to $-\nabla z(x,y)$.

First we compute $-\nabla z(x,y)$:

$$-\nabla z(x,y) = \left(\frac{2x}{\sqrt{16-4x^2-y^2}}, \frac{\frac{1}{2}y}{\sqrt{16-4x^2-y^2}} \right)$$

Therefore, it is sufficient to consider the vector $(2x, \frac{1}{2}y)$ as a tangent vector of the flow of the raindrop.

Hence we are looking for a curve $\gamma(t) = (\gamma_1(t), \gamma_2(t), z(\gamma_1(t), \gamma_2(t)))$ satisfying $\gamma_1'(t) = 2\gamma_1(t)$ and $\gamma_2'(t) = \frac{1}{2}\gamma_2(t)$.

Solving the above two equations, we have

$$\frac{d\gamma_1(t)}{dt} = 2\gamma_1(t) \qquad\qquad \frac{d\gamma_2(t)}{dt} = \frac{1}{2}\gamma_2(t)$$

$$\int \frac{1}{\gamma_1(t)} d\gamma_1(t) = 2\int dt \qquad\qquad \int \frac{1}{\gamma_2(t)} dt = \frac{1}{2}\int dt$$

$$\log|\gamma_1(t)| = 2t + C' \qquad\qquad \log|\gamma_2(t)| = \frac{1}{2}t + C''$$

$$|\gamma_1(t)| = e^{C'}e^{2t} \qquad\qquad |\gamma_2(t)| = e^{C''}e^{\frac{1}{2}t}$$

$$\gamma_1(t) = C_1 e^{2t}, \ C_1 \in \mathbb{R}. \qquad\qquad \gamma_2(t) = C_2 e^{\frac{1}{2}t}, \ C_2 \in \mathbb{R}.$$

$$\therefore \gamma(t) = \left(C_1 e^{2t}, C_2 e^{\frac{1}{2}t}, \frac{1}{2}\sqrt{16 - 4C_1^2 e^{4t} - C_2^2 e^{t}} \right), \ C_1, C_2 \in \mathbb{R} \ //$$

16. $x^3+y^3+z^3=7$, $(x_0,y_0,z_0)=(0,-1,2)$

Put $f(x,y,z)=x^3+y^3+z^3$

Then, $\nabla f(x_0,y_0,z_0) = (3x^2, 3y^2, 3z^2)\Big|_{(x,y,z)=(0,-1,2)} = (0, 3, 12)$

Thus, the tangent plane to the surface $x^3+y^3+z^3=7$ at $(0,-1,2)$ can be given by

$$\nabla f(0,-1,2)\cdot((x,y,z)-(0,-1,2))=0$$

$$\Leftrightarrow \quad (0,3,12)\cdot(x, y+1, z-2)=0$$

$$\Leftrightarrow \quad 3(y+1)+12(z-2)=0$$ //

20. S: $x^2-2y^2+5xz=7$

$(x_0,y_0,z_0)=(-1,0,-6/5)$

(a) $z=\dfrac{7-x^2+2y^2}{5x}$ Put $f(x,y)=\dfrac{7-x^2+2y^2}{5x}$

Tangent plane: $z=f(-1,0)+f_x(-1,0)(x+1)+f_y(-1,0)(y-0)$

$$= -\frac{6}{5}+\frac{-2x(5x)-(7-x^2+2y^2)5}{25x^2}\Big|_{(x,y)=(-1,0)}(x+1)+\frac{4y}{5x}\Big|_{(x,y)=(-1,0)}y$$

$$= -\frac{6}{5}+\frac{-10-(7-1)5}{25}(x+1)+0$$

$$= -\frac{6}{5}-\frac{8}{5}(x+1) = -\frac{8}{5}x-\frac{14}{5}$$ //

(b) Put $g(x,y,z)=x^2-2y^2+5xz$

Tangent plane: $\nabla g(-1,0,-6/5)\cdot(x+1, y, z+6/5)=0$

$$(2x+5z, -4y, 5x)\Big|_{(x,y,z)=(-1,0,-6/5)}\cdot(x+1, y, z+6/5)=0$$

$$(-2-6, 0, -5)\cdot(x+1, y, z+6/5)=0$$

$$-8(x+1)-5z-6=0$$

$$z=-\frac{8}{5}x-\frac{8}{5}-\frac{6}{5}=-\frac{8}{5}x-\frac{14}{5}$$ //

26. S: $x^2+4y^2=z^2$ Put $f(x,y,z)=x^2+4y^2-z^2$.

(a) Tangent plane at $(3,-2,-5)$: $\nabla f(3,-2,-5)\cdot(x-3,y+2,z+5)=0$

$$(2x,8y,-2z)\Big|_{(x,y,z)=(3,-2,-5)}\cdot(x-3,y+2,z+5)=0$$

$$(6,-16,10)\cdot(x-3,y+2,z+5)=0$$

$$6(x-3)-16(y+2)+10(z+5)=0$$

$$6x-18-16y-32+10z+50=0$$

$$6x-16y+10z=0$$

$$z=-\frac{3}{5}x+\frac{8}{5}y \quad //$$

(b) $\nabla f(0,0,0)=(2x,8y,-2z)\Big|_{(x,y,z)=(0,0,0)}=(0,0,0)$

This means that we cannot find a nonzero vector perpendicular to S at the origin.
In fact, S does not have any tangent plane at the origin.