[2.1 : 34, 36, 38, 42]

34. $x^2 + xy - xz = 2.$

(a) We can consider the function $F(x, y, z) = x^2 + xy - xz$.

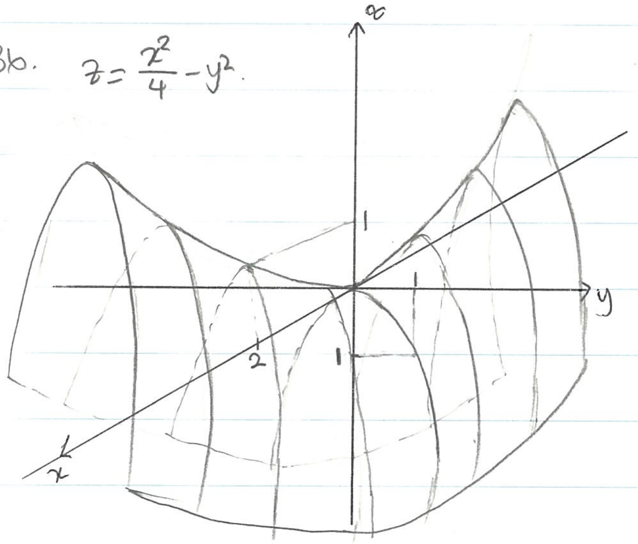
Then, the surface $x^2 + xy - xz = 2$ is a level set of $F(x, y, z)$ at height 2.

(b). Consider the surface $x^2 + xy - xz = 2$, we know that it does not include $x = 0$.

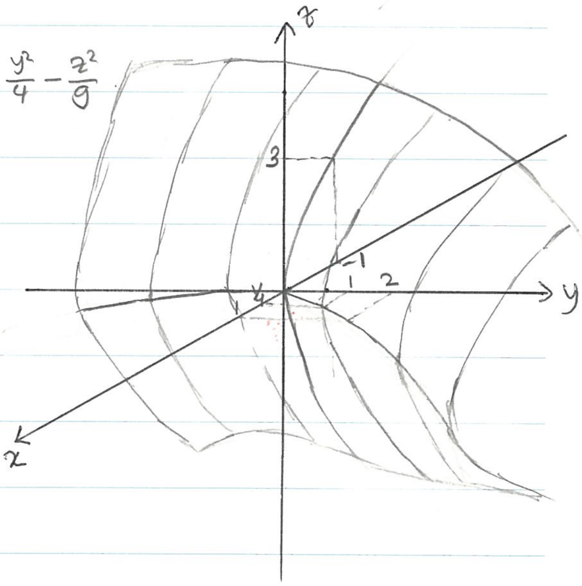
In other words, $x \neq 0$ on this surface ($\because$ when $x = 0$, $x^2 + xy - xz = 0 \neq 2$ $\square$).

Thus $x^2 + xy - xz = 2 \iff z = x + y - \frac{2}{x}$.

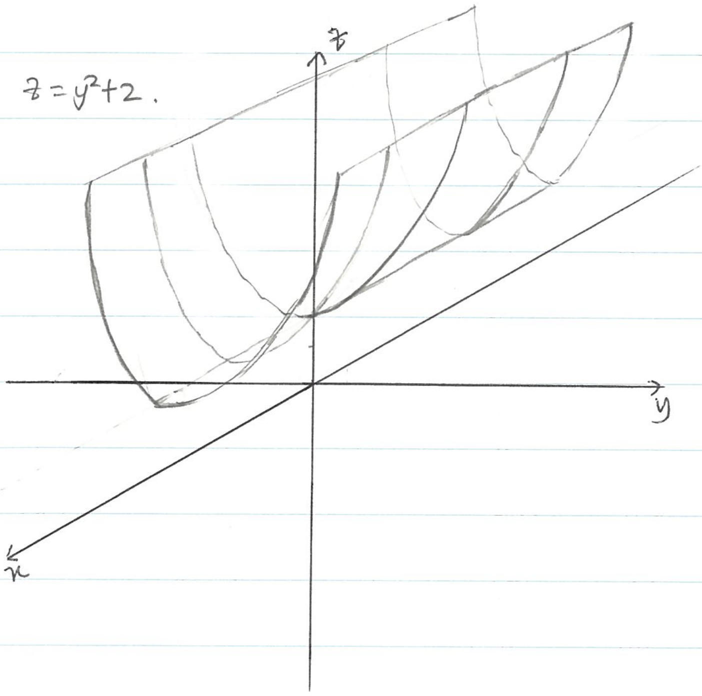
36. $z = \frac{x^2}{4} - y^2.$




38. $x = \frac{y^2}{4} - \frac{z^2}{9}.$




42. $z = y^2 + 2.$



[2.2 : 2, 3, 8, 10, 14, 16, 22, 30, 34, 36].

2. $\{(x,y) \in \mathbb{R}^2 \mid 1 \le x^2+y^2 \le 4\}$ is closed.

3. $\{(x,y) \in \mathbb{R}^2 \mid 1 \le x^2+y^2 < 4\}$ is neither open nor closed.

8. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{|y|}{\sqrt{x^2+y^2}}$ doesn't exist.

$\because$) $\displaystyle\lim_{\substack{x=ky,\,k\neq 0\\ y\to 0}} \frac{|y|}{\sqrt{x^2+y^2}} = \lim_{y\to 0} \frac{|y|}{\sqrt{k^2y^2+y^2}} = \lim_{y\to 0} \frac{|y|}{|k|\,|y|} = \frac{1}{|k|} = \begin{cases} 1, & k=1 \\ \frac{1}{2}, & k=2 \end{cases}$

That is, taking the limit of the function as $(x,y)\to(0,0)$ through the line $x=y$ and $x=2y$ gives different results. If the limit exists, no matter on the approaching method, the limit value must be the same. Hence the limit as $(x,y)\to(0,0)$ does not exist. □

10. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^x e^y}{x+y+2} = \frac{1}{2}$ //

14. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ doesn't exist.

$\because$) $\displaystyle\lim_{\substack{x=ky\\ y\to 0}} \frac{xy}{x^2+y^2} = \lim_{y\to 0} \frac{ky^2}{k^2y^2+y^2} = \lim_{y\to 0} \frac{k}{k^2+1} = \frac{k}{k^2+1} = \begin{cases} 0, & k=0, \\ \frac{1}{2}, & k=1. \end{cases}$

The limit as $(x,y)\to(0,0)$ when approaching through the lines $x=0$ and $x=y$ differs. Thus the limit does not exist. □

16. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$ doesn't exist.

$\because$) $\displaystyle\lim_{\substack{y=kx\\ x\to 0}} \frac{x^2}{x^2+y^2} = \lim_{x\to 0} \frac{x^2}{x^2+k^2x^2} = \lim_{x\to 0} \frac{1}{1+k^2} = \frac{1}{1+k^2} = \begin{cases} 1, & k=0, \\ \frac{1}{2}, & k=1. \end{cases}$

The limit as $(x,y)\to(0,0)$ when approaching through the lines $y=0$ and $y=x$ differs. Thus, the limit does not exist. □

22. (a) $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ (see Calculus 1).

(b) $\lim_{(x,y)\to(0,0)} \frac{\sin(x+y)}{x+y} = 1$

∵) Put $t(x,y) = x+y$, then $\lim_{(x,y)\to(0,0)} t(x,y) = \lim_{(x,y)\to(0,0)} (x+y) = \lim_{x\to 0} x + \lim_{y\to 0} y = 0$.

Hence, $t \to 0$ as $(x,y) \to (0,0)$. Thus applying (a), we have

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x+y)}{x+y} = \lim_{t\to 0} \frac{\sin t}{t} = 1.$$

□

(c) $\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} = 1$

∵) Put $t(x,y) = xy$, then $\lim_{(x,y)\to(0,0)} t(x,y) = \lim_{(x,y)\to(0,0)} xy = \lim_{x\to 0} x \lim_{y\to 0} y = 0$.

That is, $t \to 0$ as $(x,y) \to (0,0)$. Thus again, applying (a), we have

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} = \lim_{t\to 0} \frac{\sin t}{t} = 1.$$

□

30. $\lim_{(x,y)\to(0,0)} \frac{x^2+xy+y^2}{x^2+y^2} = \lim_{r\to 0} \frac{r^2\cos^2\theta + r^2\sin\theta\cos\theta + r^2\sin^2\theta}{r^2\cos^2\theta + r^2\sin^2\theta}$

$= \lim_{r\to 0} \frac{r^2 + r^2\frac{1}{2}\sin 2\theta}{r^2} = \lim_{r\to 0}\left(1+\frac{1}{2}\sin 2\theta\right) = 1+\frac{1}{2}\sin 2\theta$

The limit depends on $\theta$ which means according from where we approach the origin the limit varies, for example, if we approach from $\theta=0$ and $\theta=\frac{\pi}{4}$, we have $\lim_{(x,y)\to(0,0)} \frac{x^2+xy+y^2}{x^2+y^2} = \begin{cases} 1, & \theta=0 \\ \frac{3}{2}, & \theta=\frac{\pi}{4} \end{cases}$

Thus the limit does not exist.

34. For any $(a,b)\in\mathbb{R}^2$,

$\lim_{(x,y)\to(a,b)} f(x,y) = \lim_{(x,y)\to(a,b)} (x^2+2xy-y^7) = \lim_{x\to a} x^2 + 2\lim_{(x,y)\to(a,b)} xy - \lim_{y\to b} y^7$

$= \lim_{x\to a} x \lim_{x\to a} x + 2\lim_{x\to a} x \lim_{y\to b} y - \left(\lim_{y\to b} y\right)^7$

$= a^2+2ab-b^7 = f(a,b)$

(Here we have used some properties of limits since we know that $\lim_{x\to a} x$ and $\lim_{y\to b} y$ exist.)

Thus, $f(x,y)$ is continuous everywhere on $\mathbb{R}^2$.

36. $g(x,y) = \frac{x^2-y^2}{x^2+1}$ is defined everywhere on $\mathbb{R}^2$ since $x^2+1 \neq 0$ for all $x\in\mathbb{R}$.

For any $(a,b)\in\mathbb{R}^2$,

$\lim_{(x,y)\to(a,b)} g(x,y) = \lim_{(x,y)\to(a,b)} \frac{x^2-y^2}{x^2+1} = \lim_{(x,y)\to(a,b)} x^2-y^2 \lim_{x\to a} \frac{1}{x^2+1}$

$= \left(\lim_{x\to a} x^2 - \lim_{y\to b} y^2\right)\cdot\frac{1}{a^2+1} = \frac{a^2-b^2}{a^2+1} = g(a,b)$

(Here we also used some properties of limits since we know that $\lim_{x\to a} x$, $\lim_{y\to b} y$, $\lim_{x\to a}\frac{1}{x^2+1}$ exist.)

Thus, $g(x,y)$ is continuous everywhere on $\mathbb{R}^2$.