[1.7 : 2, 4, 8, 18, 22, 26, 34]

2. $r = \sqrt{3}, \quad \theta = \frac{5\pi}{6}$.

$x = r\cos\theta = \sqrt{3}\cos\frac{5\pi}{6} = \sqrt{3}\cdot\left(-\frac{1}{2}\sqrt{3}\right) = -\frac{3}{2}$.

$y = r\sin\theta = \sqrt{3}\sin\frac{5\pi}{6} = \sqrt{3}\cdot\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{3}$.

The Cartesian coordinates of $(\sqrt{3}, \frac{5\pi}{6})$ are $(-\frac{3}{2}, \frac{1}{2}\sqrt{3})$ //

4. $x = 2\sqrt{3}, \quad y = 2$.

$r^2 = x^2 + y^2 = 16 \Rightarrow r = 4$.

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{3}\sqrt{3}\right) = \frac{\pi}{6}$.

The polar coordinates of $(2\sqrt{3}, 2)$ are $(4, \frac{\pi}{6})$ //

8. $r = \pi, \quad \theta = \frac{\pi}{2}, \quad z = 1$.

$x = r\cos\theta = \pi\cos\frac{\pi}{2} = \pi \cdot 0 = 0$.

$y = r\sin\theta = \pi\sin\frac{\pi}{2} = \pi \cdot 1 = \pi$.

The Cartesian coordinates of $(\pi, \frac{\pi}{2}, 1)$ are $(0, \pi, 1)$ //

18. $x = 0, \quad y = \sqrt{3}, \quad z = 1$.

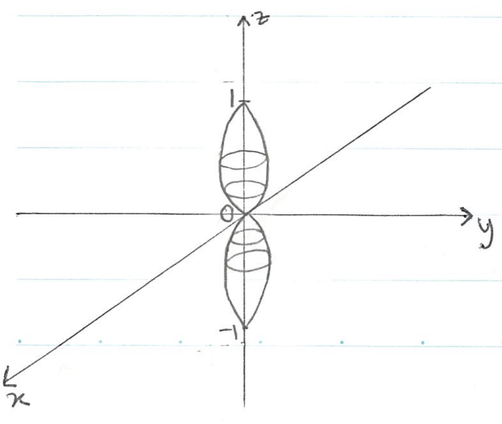
$\rho^2 = x^2 + y^2 + z^2 = 3 + 1 = 4 \Rightarrow \rho = 2$.

$\varphi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$.

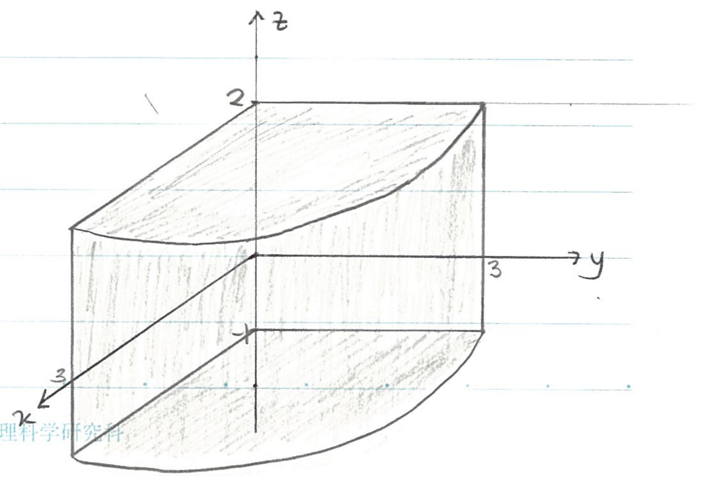
$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\infty = \frac{\pi}{2}$.

The spherical coordinates of $(0, \sqrt{3}, 1)$ are $(2, \frac{\pi}{3}, \frac{\pi}{2})$ //

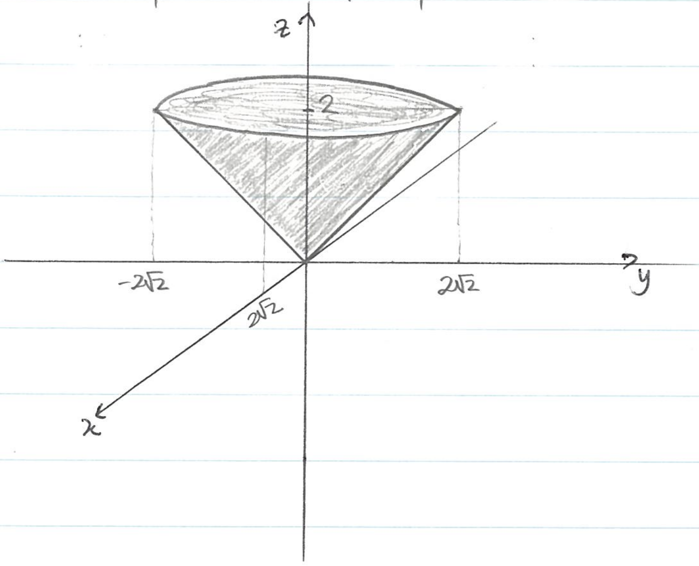
22. $\rho = 1 - \sin\varphi$.



26. $0 \le r \le 3, \quad 0 \le \theta \le \frac{\pi}{2}, \quad -1 \le z \le 2$.



34. $0 \le \rho \le \frac{2}{\cos\varphi}$, $0 \le \varphi \le \frac{\pi}{4}$.



[2.1 : 2, 4, 10, 12, 14, 24]

2. $g : \mathbb{R}^2 \longrightarrow \mathbb{R}$

$(x,y) \longmapsto 2x^2+3y^2-7$.

(a) Since $2x^2+3y^2-7$ is defined for all $x,y \in \mathbb{R}$, $\mathbb{R}^2$ is the domain of g //

Since $2x^2+3y^2 \geq 0$, $2x^2+3y^2-7 \geq -7$ for all $x,y \in \mathbb{R}$.

Thus $\{x \in \mathbb{R} \mid x \geq -7\}$ is the range of g. //

$= \{(x,y) \in \mathbb{R} \mid x \geq 0, y \geq 0\}$.

(b) We may restrict the domain to $\mathbb{R}^2 \setminus \{(x,y) \in \mathbb{R}^2 \mid x<0 \text{ or } y<0\}$, since

if $x \geq 0$, $\sqrt{x^2} = |x| = x$ is defined by one value and so $\sqrt{y^2} = |y| = y$.

Thus for any value x^2 any y^2, only $x = |x|$ and $y = |y|$ correspond to $2x^2+3y^2$.

Since a linear polynomial is one-one, we find that

$g : \mathbb{R}^2 \setminus \{(x,y) \in \mathbb{R}^2 \mid x<0 \text{ or } y<0\} \longrightarrow \mathbb{R}$

$(x,y) \longmapsto 2x^2+3y^2-7$

is one-one //

(c) Since $\{x \in \mathbb{R} \mid x \geq -7\}$ is the range of g, restricting $\mathbb{R}$ to $\{x \in \mathbb{R} \mid x \geq -7\}$ as

$g : \mathbb{R}^2 \longrightarrow \{x \in \mathbb{R} \mid x \geq -7\}$

$(x,y) \longmapsto 2x^2+3y^2-7$,

gives us an onto function g.

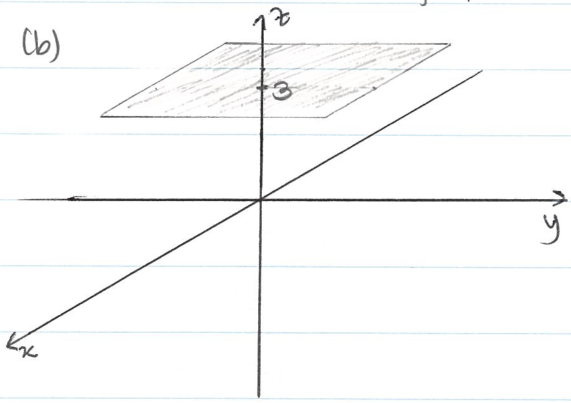
4. $f(x,y) = \ln(x+y)$

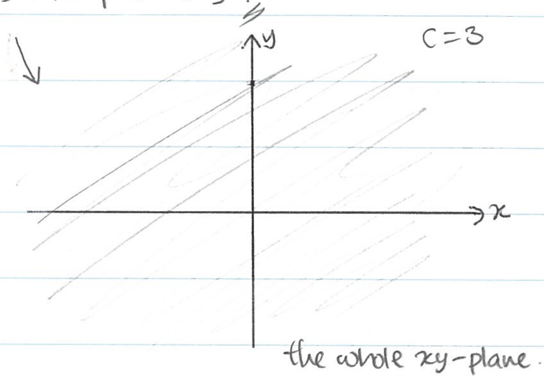
f is only defined for $x+y>0$. Thus $\{(x,y)\in\mathbb{R}^2 \mid x+y>0\}$ is the domain of f.
$\ln x$ is an onto function defined on $\mathbb{R}_{>0} = \{x\in\mathbb{R} \mid x>0\}$ with inverse function $\exp(y)$ which is defined for all $y\in\mathbb{R}$. Thus $\ln: \mathbb{R}_{>0} \to \mathbb{R}$ is an onto function.
Therefore, $\mathbb{R}$ is the range of f.

10. $f(x,y) = 3$.

(a) Since f is a constant function, f has level curve only at its value, which is 3.
Thus, the level curve of f at height $c = \{(x,y)\in\mathbb{R}^2 \mid c=3\}$

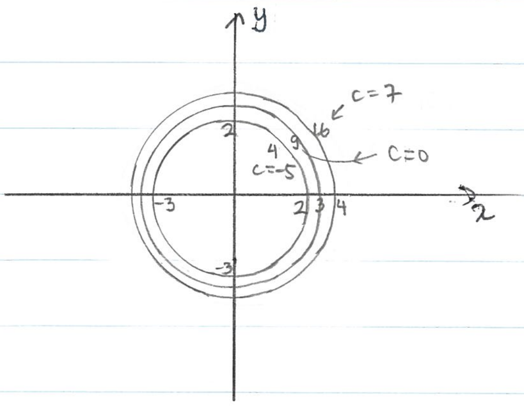
(b)



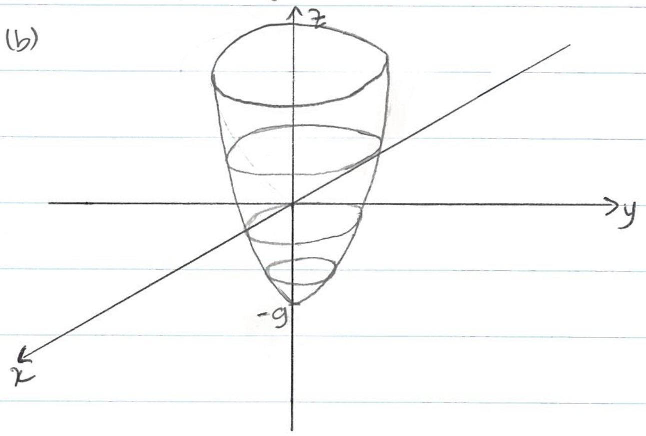


12. $f(x,y) = x^2+y^2-9$.

(a) Level curve of f at height $c = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2-9=c\} = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2=9+c\}$ //



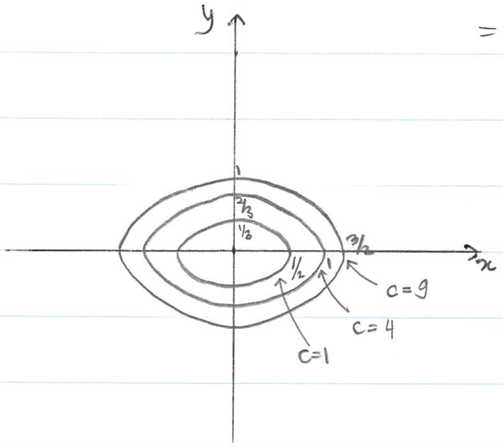
(b)



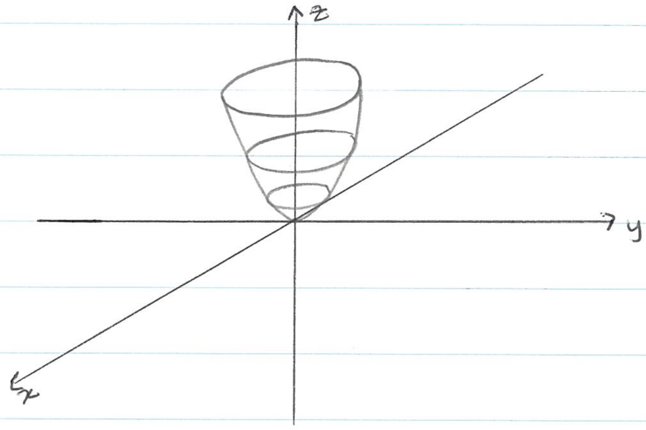
14. $f(x,y) = 4x^2+9y^2$

(a) Level curve of f at height $c = \{ (x,y) \in \mathbb{R}^2 \mid 4x^2+9y^2 = c \}$

$$= \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; \left(\frac{x}{\sqrt{c}/2}\right)^2 + \left(\frac{y}{\sqrt{c}/3}\right)^2 = 1 \right\}$$

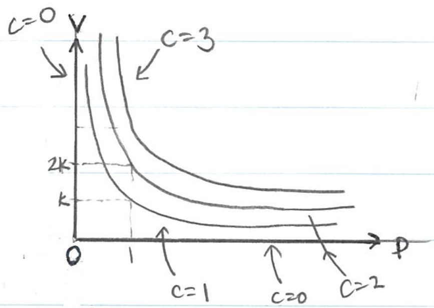


(b)



21. $PV = kT$.

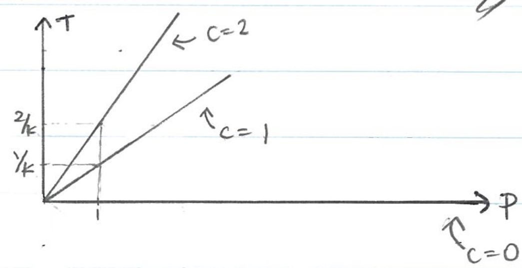
(a) $T = \frac{1}{k} PV$.



Level curve of T at height c

$$= \{ (P,V) \in \mathbb{R}^2_{\geq 0} \mid \tfrac{1}{k} PV = c \} = \{ (P,V) \in \mathbb{R}^2_{\geq 0} \mid PV = kc \}$$

(b) $V = k \frac{T}{P}$



Level curve of V at height c

$$= \{ (P,T) \in \mathbb{R}^2_{\geq 0} \mid k\tfrac{T}{P} = c \} = \{ (P,T) \in \mathbb{R}^2_{\geq 0} \mid \tfrac{T}{P} = \tfrac{c}{k} \}$$