[1.3 : 16, 20, 24]

16. We want to find $\vec{v}_1, \vec{v}_2, \vec{v}_3$ s.t. $\vec{v}_i \not\parallel \vec{v}_j$ when $i \neq j$ $(i,j=1,2,3)$ and $\vec{v}_i \perp (\hat{\imath}-\hat{\jmath}+\hat{k})$ for $i=1,2,3$.

Writing $\vec{v}_i = a_i\hat{\imath} + b_i\hat{\jmath} + c_i\hat{k}$, then we must have

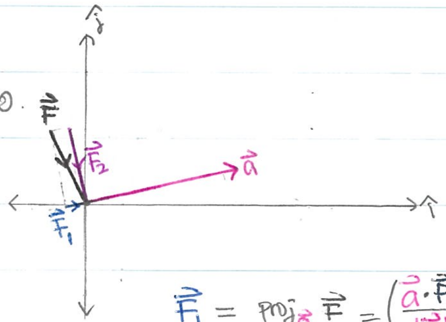
$$(a_i\hat{\imath}+b_i\hat{\jmath}+c_i\hat{k})\cdot(\hat{\imath}-\hat{\jmath}+\hat{k}) = a_i - b_i + c_i = 0 \quad \text{for all } i=1,2,3 \;\cdots\; (*)$$

We can take $\vec{v}_1 = \hat{\imath}+\hat{\jmath}$, $\vec{v}_2 = \hat{\jmath}+\hat{k}$, and $\vec{v}_3 = \hat{\imath}-\hat{k}$, for we can check easily that $(*)$ is satisfied. We can also check that $\vec{v}_i \not\parallel \vec{v}_j$ when $i\neq j$ $(i,j=1,2,3)$ by showing that $\vec{v}_i$ and $\vec{v}_j$ are linearly independent when $i \neq j$ for $i,j=1,2,3$:

$$\begin{cases} c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{0} \iff c_1\hat{\imath} + (c_1+c_2)\hat{\jmath} + c_2\hat{k} = \vec{0} \iff c_1=c_2=0 \\ c_1\vec{v}_1 + c_3\vec{v}_3 = \vec{0} \iff (c_1+c_3)\hat{\imath} + c_1\hat{\jmath} - c_3\hat{k} = \vec{0} \iff c_1=c_3=0. \\ c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0} \iff c_3\hat{\imath} + c_2\hat{\jmath} + (c_2-c_3)\hat{k} = \vec{0} \iff c_2=c_3=0. \end{cases}$$

$\square$

20.



$\vec{F} = \hat{\imath} - 2\hat{\jmath}$

$\vec{a} = 4\hat{\imath} + \hat{\jmath}$.

We want to find $\vec{F_1}$ and $\vec{F_2}$ such that $\vec{F} = \vec{F_1} + \vec{F_2}$ (see figure).

$$\vec{F_1} = \mathrm{proj}_{\vec{a}}\vec{F} = \left(\frac{\vec{a}\cdot\vec{F}}{\|\vec{a}\|^2}\right)\vec{a} = \frac{4-2}{17}(4\hat{\imath}+\hat{\jmath}) = \frac{2}{17}(4\hat{\imath}+\hat{\jmath}) = \frac{8}{17}\hat{\imath} + \frac{2}{17}\hat{\jmath}$$

$$\vec{F_2} = \vec{F} - \vec{F_1} = (\hat{\imath}-2\hat{\jmath}) - \left(\frac{8}{17}\hat{\imath} + \frac{2}{17}\hat{\jmath}\right) = \frac{9}{17}\hat{\imath} - \frac{36}{17}\hat{\jmath} = \frac{9}{17}(\hat{\imath}-4\hat{\jmath})$$

24. Consider $A(a_1,a_2,a_3)$, $B(b_1,b_2,b_3)$, $C(c_1,c_2,c_3)$, $D(d_1,d_2,d_3) \in \mathbb{R}^3$.

Let M_1, M_2, M_3, and M_4 be the midpoints of $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ respectively.

Then we have $M_1\left(\frac{b_1+a_1}{2}, \frac{b_2+a_2}{2}, \frac{b_3+a_3}{2}\right)$, $M_2\left(\frac{c_1+b_1}{2}, \frac{c_2+b_2}{2}, \frac{c_3+b_3}{2}\right)$, $M_3\left(\frac{d_1+c_1}{2}, \frac{d_2+c_2}{2}, \frac{d_3+c_3}{2}\right)$, and $M_4\left(\frac{a_1+d_1}{2}, \frac{a_2+d_2}{2}, \frac{a_3+d_3}{2}\right)$.

We find that

$$\begin{cases} \overrightarrow{M_1M_2} \parallel \overrightarrow{M_3M_4} \text{ since } \overrightarrow{M_1M_2} = \frac{c_1-a_1}{2}\hat{\imath} + \frac{c_2-a_2}{2}\hat{\jmath} + \frac{c_3-a_3}{2}\hat{k} = -\left(\frac{a_1-c_1}{2}\hat{\imath} + \frac{a_2-c_2}{2}\hat{\jmath} + \frac{a_3-c_3}{2}\hat{k}\right) \\ \qquad = -\overrightarrow{M_3M_4}. \\ \overrightarrow{M_2M_3} \parallel \overrightarrow{M_4M_1} \text{ since } \overrightarrow{M_2M_3} = \frac{d_1-b_1}{2}\hat{\imath} + \frac{d_2-b_2}{2}\hat{\jmath} + \frac{d_3-b_3}{2}\hat{k} = -\left(\frac{b_1-d_1}{2}\hat{\imath} + \frac{b_2-d_2}{2}\hat{\jmath} + \frac{b_3-d_3}{2}\hat{k}\right) \\ \qquad = -\overrightarrow{M_4M_1}. \end{cases}$$

[1.4 : 10, 12, 18]

10. Consider a parallelogram with vertices $A(1,1)$, $B(3,2)$, $C(1,3)$, $D(-1,2)$.

Then the area is $\| \overrightarrow{AB} \times \overrightarrow{AC} \| = \| (2\hat{i} + \hat{j}) \times (2\hat{j}) \| = \| 4\hat{k} \| = 4$ units area //

12. $\vec{v} = (2\hat{i} + \hat{j} - 3\hat{k}) \times (\hat{i} + \hat{k}) = \begin{vmatrix} 1 & -3 \\ 0 & 1 \end{vmatrix} \hat{i} - \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} \hat{j} + \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} \hat{k} = \hat{i} - 5\hat{j} - \hat{k}$.

The unit vector we want to find is $\dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{\hat{i} - 5\hat{j} - \hat{k}}{\sqrt{1+25+1}} = \dfrac{1}{\sqrt{27}}(\hat{i} - 5\hat{j} - \hat{k})$ //

18. The volume of the parallelepiped is $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |(-\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (\hat{i} - 2\hat{j} + 4\hat{k})|$

$= |-1+6-8| = 3$ units volume //

[1.5 : 6, 12, 16, 20, 23, 26]

6. Since the plane contains the line $x = 2t-1$, $y = 3t+4$, $z = 7-t$, for $t=0$ and $t=1$, we find that the plane also contains the points $A(-1,4,7)$ and $B(1,7,6)$, besides the point $C(2,5,0)$.

Thus a normal vector $\vec{n}$ of the plane is $\overrightarrow{AB} \times \overrightarrow{AC} = -20\hat{i} + 11\hat{j} - 7\hat{k}$.

The plane can then be expressed as $-20(x-2) + 11(y-5) - 7z = 0$. //

12. A normal vector of the first plane is $A\hat{i} - \hat{j} + \hat{k}$ and that of the second plane is $3A\hat{i} + A\hat{j} - 2\hat{k}$.

The two planes are perpendicular to each other if and only if their normal vectors are.

Thus, $(A\hat{i} - \hat{j} + \hat{k}) \cdot (3A\hat{i} + A\hat{j} - 2\hat{k}) = 0$ which gives us $3A^2 - A - 2 = 0$.

Solving the equation for A, we have $A = 1$ or $A = -\frac{2}{3}$ //

16. Since the plane passes the points $A(0,2,1)$, $B(7,-1,5)$, $C(-1,3,0)$, we have

$\vec{x}(s,t) = s\overrightarrow{AB} + t\overrightarrow{AC} + \overrightarrow{OA} = s(7\hat{i} - 3\hat{j} + 4\hat{k}) + t(-\hat{i} + \hat{j} - \hat{k}) + 2\hat{j} + \hat{k}$

$= (7s-t)\hat{i} + (-3s+t+2)\hat{j} + (4s-t+1)\hat{k}$,

i.e., $\begin{cases} x = 7s-t \\ y = -3s+t+2 \\ z = 4s-t+1 \end{cases}$

20. $P_0(1,-2,3)$, $\vec{l}(t) = t(2,-1,0) + (-5,3,4)$. We have $\vec{a} = (2,-1,0)$ and $B(-5,3,4)$.

$$D = \|\overrightarrow{BP_0} - \mathrm{proj}_{\vec{a}}\,\overrightarrow{BP_0}\| = \|(6,-5,-1) - \left(\frac{(2,-1,0)\cdot(6,-5,-1)}{4+1}\right)(2,-1,0)\|$$

$$= \|(6,-5,-1) - \frac{12+5}{5}(2,-1,0)\| = \|(6,-5,-1) - \frac{17}{5}(2,-1,0)\| = \|-\frac{1}{5}(4,8,5)\|$$

$$= \frac{1}{5}\sqrt{105} \text{ units.}$$

23. $\vec{l_1}(t) = t(8,-1,0) + (-1,3,5)$, $\vec{l_2}(t) = t(0,3,1) + (0,3,4)$.

$=: \vec{a_1}t + B_1$ $\qquad$ $=: \vec{a_2}t + B_2$.

$$\vec{n} = \vec{a_1}\times\vec{a_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -1 & 0 \\ 0 & 3 & 1 \end{vmatrix} = -\hat{i} - 8\hat{j} + 24\hat{k}.$$

$$D = \|\mathrm{proj}_{\vec{n}}\,\overrightarrow{B_1B_2}\| = \left\|\frac{\vec{n}\cdot\overrightarrow{B_1B_2}}{\|\vec{n}\|^2}\vec{n}\right\| = \left\|\frac{(-1,-8,24)\cdot(1,0,-1)}{1+64+576}(-1,-8,24)\right\|$$

$$= \left\|\frac{-1-24}{641}(-1,-8,24)\right\| = \left\|\frac{-25}{641}(-1,-8,24)\right\| = \frac{25}{\sqrt{641}} \text{ units.}$$

26. (a) $\vec{l_1}(t) = t(1,-1,5) + (2,0,-4) =: \vec{a_1}t + B_1$

$\vec{l_2}(t) = t(1,-1,5) + (1,3,-5) =: \vec{a_2}t + B_2$.

$\vec{l_1}(t)$ and $\vec{l_2}(t)$ are parallel for $\vec{a_1} = \vec{a_2}$. Thus we cannot find a vector perpendicular to both lines by taking the cross product of $\vec{a_1}$ and $\vec{a_2}$.

Therefore, the method of Example 9 fails.

(b) We can consider this case as finding the distance between the point B_1 and the line $\vec{l_2}(t)$.

Then, $D = \|\overrightarrow{B_2B_1} - \mathrm{proj}_{\vec{a_2}}\,\overrightarrow{B_2B_1}\| = \|(1,-3,1) - \frac{\vec{a_2}\cdot\overrightarrow{B_2B_1}}{\|\vec{a_2}\|^2}\vec{a_2}\|$

$$= \|(1,-3,1) - \frac{(1,-1,5)\cdot(1,-3,1)}{1+1+25}(1,-1,5)\| = \|(1,-3,1) - \frac{1+3+5}{27}(1,-1,5)\|$$

$$= \|(1,-3,1) - \frac{1}{3}(1,-1,5)\| = \|(\frac{2}{3}, -\frac{8}{3}, -\frac{2}{3})\| = \sqrt{\frac{4+64+4}{9}} = \sqrt{8} = 2\sqrt{2} \text{ units}$$