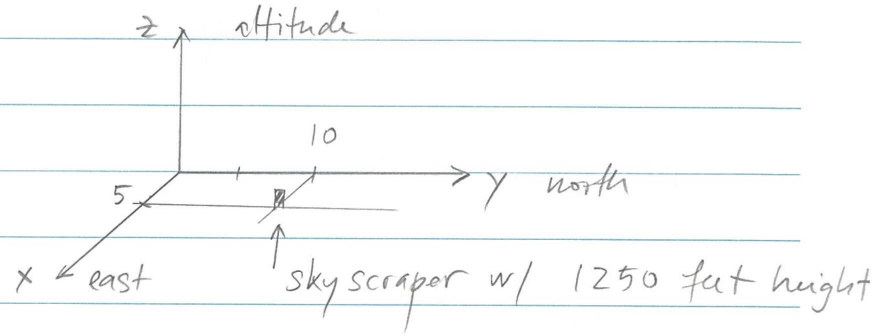
Homework 1 Solutions

§1.1. #16 $A = (2, 5, -6)$, $\vec{AB} = (12, -3, 7)$ are given, where $B = (B_1, B_2, B_3)$ is unknown.
Then, since $\vec{AB} = (B_1 - A_1, B_2 - A_2, B_3 - A_3)$ it follows that $12 = B_1 - 2$, $-3 = B_2 - 5$, $7 = B_3 + 6$

$$\Rightarrow B_1 = 14, \quad B_2 = 2, \quad B_3 = 1 \quad \Rightarrow \underline{B = (14, 2, 1)}$$

#24 $\vec{v} = (50, 100, 4)$ m/h



(a) The plane is climbing at a rate of 4 m/h.

(b) Let t be time measured in hours. 1 mile = 5280 ft.
Then the position of the airplane is given by

$x(t) = 50t$
$y(t) = 100t$ because $(x(0), y(0), z(0)) = 0$
$z(t) = 4t$ and $\vec{v}(t) = \vec{v} = (50, 100, 4)$

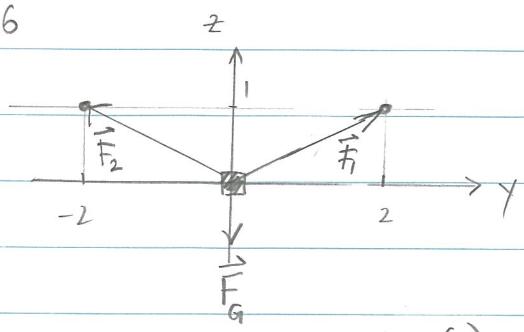
Note: $x(0.1) = 5$, $y(0.1) = 10$ $\Rightarrow$ after 0.1 hour (= 6 min) the plane is on top of the skyscraper

(c) The height of the airplane at $t = 0.1$ min is $z(0.1) = 0.4$ miles

$$\rightarrow z(0.1) = \frac{4}{10} \cdot 5280 \text{ ft} = 4 \cdot 528 = 2112$$

$\Rightarrow$ The vertical clearance is $2112 - 1250 = 862$ ft.

#26



$\vec{F}_G = (0,0,-1)mg$

m = mass, g gravitational constant $9.81\, m/s^2$

$\Rightarrow$ (a) $m = 50$ lbs $\approx 50 \cdot 0.454$ kg $= 22.7$ kg

$\Rightarrow \vec{F}_G \approx (0,0,-222.7)\ m\cdot kg/s^2$

(b) $\vec{F}_1 = a(0,+2,1)$, $\vec{F}_2 = a(0,-2,1)$ (b/c of symmetry)

$\vec{F}_1 + \vec{F}_2 + \vec{F}_G = 0 \Rightarrow 2a - 222.7 = 0 \Rightarrow a = 111.35$

$\Rightarrow \vec{F}_1 = \frac{mg}{2}(0,+2,1)$, $\vec{F}_2 = \frac{mg}{2}(0,-2,1)$

$\approx 111.35\,(0,+2,1)$ $\approx 111.35\,(0,-2,1)$

§1.2 #14 $P = (12,-2,0)$, line $\parallel$ to $5\vec{i} - 12\vec{j} + \vec{k}$

$\Rightarrow$ parametric eqns. for the line $\begin{cases} x(t) = 12 + 5t \\ y(t) = -2 - 12t \\ z(t) = t \end{cases}$

#16 $P = (2,1,2)$, $Q = (3,-1,5)$, then the parametric eqns.

for the line through the points is

$$\begin{cases} x(t) = P_1 + (\vec{PQ})_1 t = 2 + t \\ y(t) = P_2 + (\vec{PQ})_2 t = 1 - 2t \\ z(t) = P_3 + (\vec{PQ})_3 t = 2 + 3t \end{cases}$$

#22 The line goes through the point $(2,3,-1)$ and is parallel to $(5,-2,4)$

$\Rightarrow$ parametric eqns. $\begin{cases} x(t) = 2+5t \\ y(t) = 3-2t \\ z(t) = 4 - t \end{cases}$

#30 Given: line $\begin{cases} x(t) = 1-4t \\ y(t) = t - 3/2 \\ z(t) = 2t+1 \end{cases}$, plane : $5x-2y+z=1$

Line & plane intersect when: $5(1-4t) - 2(t-\frac{3}{2}) + (2t+1) = 1$

$\Leftrightarrow \quad 5-20t - 2t+3 + 2t+1 = 1$

$8 - 20t = 0 \Rightarrow t = \frac{8}{20} = \frac{2}{5} \Rightarrow$ the point of

intersections is $\left(x\left(\frac{2}{5}\right), y\left(\frac{2}{5}\right), z\left(\frac{2}{5}\right)\right) = \left(1-\frac{8}{5}, \frac{2}{5}-\frac{3}{2}, \frac{4}{5}+1\right)$

$= \underline{\left(-\frac{3}{5}, -\frac{11}{10}, \frac{9}{5}\right)}$

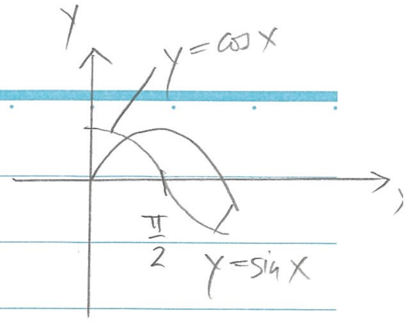
#37 (a) $x^2+y^2=4 \Rightarrow (x(t), y(t))$ is on the circle of radius 2,

$0 \le t < \frac{2\pi}{3} \Rightarrow 0 < 3t < 2\pi \Rightarrow$ the curve is the circle of radius 2, centered at the origin, traced out 1 time, counterclockwise, starting at $(2,0)$.

If $0 \le t \le 2\pi$, then the circle is traced out 3 times

(b) The circle of radius 5, centered at $(0,0)$, traced out one time, counterclockwise, starting at $(5,0)$

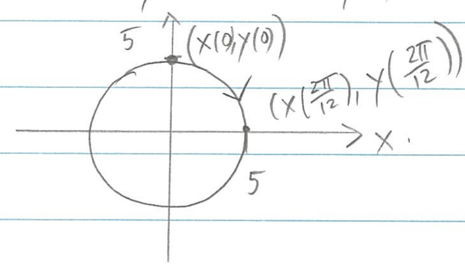
(c) The circle of radius 5, centered at the origin, traced out one time, starting at $(0,5)$, since



$$\left(x\left(\frac{2\pi}{3\cdot 4}\right), y\left(\frac{2\pi}{3\cdot 4}\right)\right) = \left(5\sin\left(\frac{\pi}{2}\right), 5\cos\left(\frac{\pi}{2}\right)\right)$$

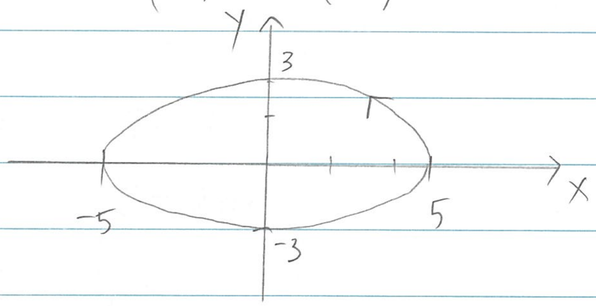
$$= (5,0)$$

$\Rightarrow$



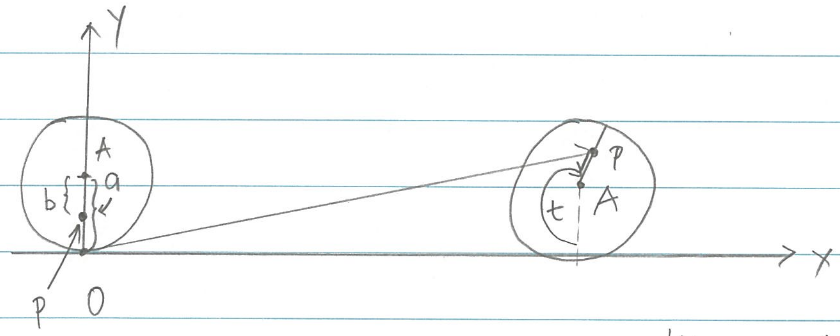
$\Rightarrow$ the circle is traced out clockwise.

(d) $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{3}\right)^2 = 1 \Rightarrow (x(t), y(t))$ describes an ellipse w/ center $(0,0)$ and



it is traced out one time, starting at $(5,0)$, going counterclockwise.

#38

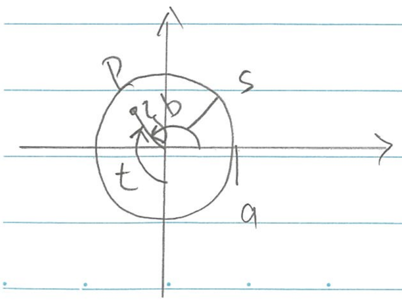


t measured in radians

like in class: $\overrightarrow{OA} = (at, a)$

Want to know: $\overrightarrow{OP}$!

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$; so what is $\overrightarrow{AP}$?



$\Rightarrow$ (1) $s + t = \frac{3\pi}{2}$

(2) $\overrightarrow{AP} = (b\cos(s), b\sin(s))$

$$\Rightarrow P = \left(b\cos\left(\frac{3\pi}{2} - t\right),\ b\sin\left(\frac{3\pi}{2} - t\right)\right)$$

$$= \left(-b\sin(t),\ -b\cos(t)\right)$$

$$\Rightarrow \underline{\vec{OP} = \vec{i}\,(at - b\sin t) + \vec{j}\,(a - b\cos(t))}$$

§1.3. #8 $\vec{a} = \vec{i} + \vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j} + 2\vec{k}$

$$\rightarrow \quad \cos\theta = \frac{\vec{a}\circ\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|} = \frac{-1+2-2}{\sqrt{3}\cdot 3} = \frac{-1}{3\sqrt{3}}$$

$\Rightarrow$ angle between the vectors $\underline{\cos^{-1}\left(\frac{-1}{3\sqrt{3}}\right)}$

#10 $\vec{a} = \vec{i} + \vec{j}$, $\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$

$\Rightarrow$ $\vec{a}\circ\vec{b} = 2 + 3 = 5$, $\|\vec{a}\| = \sqrt{2}$, $\underbrace{\|\vec{b}\|}_{\text{don't need that.}} = \sqrt{14}$

$$\mathrm{proj}_{\vec{a}}\,\vec{b} = \underline{\frac{5}{2}(\vec{i} + \vec{j})}$$