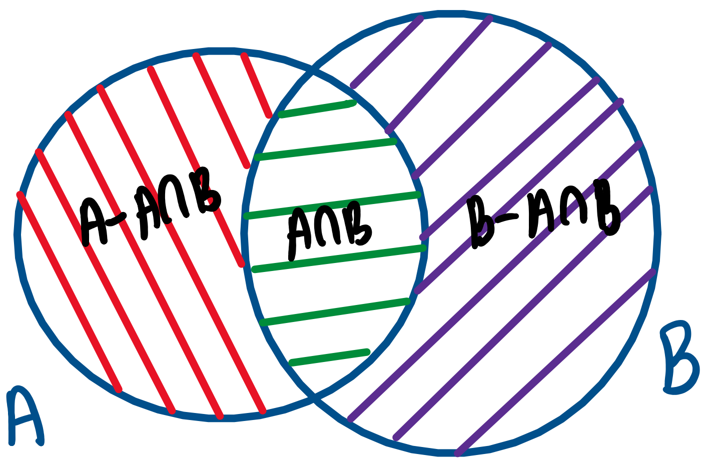
Exercise 1.1.6.

Thursday, 19 October 2023 22:21

1) $A^c \cup A = \Omega$ and $A^c \cap A = \varnothing$

$P(\Omega) = P(A^c \cup A) = P(A^c) + P(A)$ $\quad$ $P(A^c) = P(\Omega) - P(A) = 1 - P(A)$

2)



$A \cup B = A - A\cap B + A \cap B + B - A \cap B$

$(A - A\cap B) \cap (A \cap B) = \varnothing$

$(A - A\cap B) \cap (B - A\cap B) = \varnothing$ $\quad$ $P(A\cup B) = P(A - A\cap B) + P(A\cap B) + P(B - A\cap B)$ $(*)$

$(B - A\cap B) \cap (A \cap B) = \varnothing$

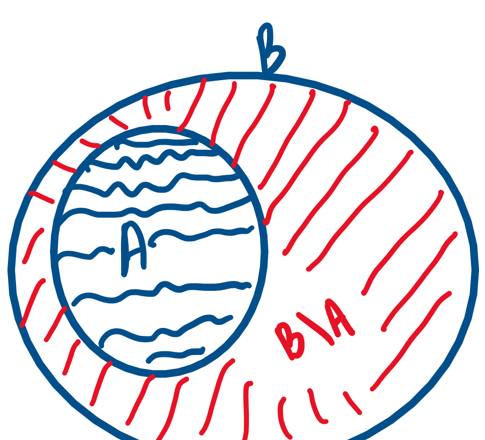
$P(A) = P((A - A\cap B) \cup (A\cap B)) = P(A - A\cap B) + P(A\cap B)$ $\quad$ $P(A - A\cap B) = P(A) - P(A\cap B)$ $(**)$

$P(B) = P((B - A\cap B) \cup (A\cap B)) = P(B - A\cap B) + P(A\cap B)$ $\quad$ $P(B - A\cap B) = P(B) - P(A\cap B)$ $(***)$

Substituting $(**)$ and $(***)$ into $(*)$

$P(A\cup B) = P(A) - P(A\cap B) + P(A\cap B) + P(B) - P(A\cap B) = P(A) + P(B) - P(A\cap B)$

3)



$B = A \cup (B\setminus A)$ $\quad$ $A \cap (B\setminus A) = \varnothing$

$P(B) = P(A \cup (B\setminus A)) = P(A) + P(B\setminus A)$

$P(B) - P(A) = P(B\setminus A)$ $\quad$ We know $P(B\setminus A) \geq 0$

Therefore, $P(B) - P(A) \geq 0 \Rightarrow P(B) \geq P(A)$