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Exercise (A First Course in Stochastic Calculus)

2.1 An example of uncorrelated random variables that are not independent.

X and X^2

If you have X and X^2 with $X \sim N(0,1)$,

then $Cov(X, X^2) = E(X^3) - E(X)E(X^2) = 0$.

=0, see next page

but the two random variables are crearly dependent. ☒

2.3 why $\sqrt{\pi}$?

$x = r\sin z,\ y = r\cos z \Rightarrow dxdy = rdrdz$

$$\therefore \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2} e^{-y^2} dxdy$$

$$= \int_0^{2\pi}\int_0^{\infty} e^{-r^2\sin^2 z} \cdot e^{-r^2\cos^2 z} \cdot r \cdot drdz$$

$$= \int_0^{2\pi}\int_0^{\infty} e^{-r^2} \cdot r \cdot drdz$$

$$= \int_0^{\infty} e^{-r^2} \cdot rdr \cdot \int_0^{2\pi} dz$$

$$= 2\pi \cdot \frac{1}{2}$$

$$= \pi$$

$$\therefore \int_{-\infty}^{\infty} \frac{e^{-x^2}}{\sqrt{\pi}} dx = 1 \ ☒$$

x^3 : odd, $e^{-\frac{x^2}{2}}$: even $\Rightarrow$ $x^3 \cdot e^{-\frac{x^2}{2}}$: odd.

Generally, if a $f(x)$ is odd, $f(-x) = -f(x)$.

$$\therefore \int_{-a}^{0} f(x)\,dx = \int_{a}^{0} f(-t)(-dt) = \int_{a}^{0} \{-f(t)\}(-dt)$$

$$= -\int_{0}^{a} f(t)\,dt = -\int_{0}^{a} f(x)\,dx$$

$$\therefore \int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= -\int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx = 0.$$

$$\therefore \mathbb{E}(X^3) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{x^3 \cdot e^{-\frac{x^2}{2}}}_{\text{odd}}\,dx = 0 \quad \square$$