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Exercise 1.1.3

When a power set Ω contains N elements,

it contains 2^N elements.

Proof

[1] If $N=1$, a power set will be $\{\phi\}, \{1\}$

So, the number of elements is $2 := 2^1$

[2] Now, we can count it deductively

The number of elements in a set Ω contains N elements,

We can think it is same as the way to choose n from N.

$$\left[\begin{array}{l} \phi \,/\, \{1\}, \cdots \{N\} \,/ \\ \{1.2\}, \cdots \{N-1.N\} \,/ \\ \quad \vdots \\ /\,\{1.2.3. \cdots N\}\,/ \end{array}\right] \qquad \underbrace{\binom{N}{0}.\ \binom{N}{1}.\ \binom{N}{2}.\ \cdots\ \binom{N}{N}}$$

The number of sets with n elements is same as the way to choose n from N.

Therefore,

$$\binom{N}{0}+\binom{N}{1}+\cdots+\binom{N}{N} = 2^N \quad \#$$

Example

When $N=4$, the elements of a set Ω can be $\{1.2.3.4\}$

Then the power set will be

$\{\phi\}.\ \{1\}.\ \{2\}.\ \{3\}.\ \{4\}$

$\{1.2\}.\ \{1.3\}.\ \{1.4\},\ \{2.3\},\ \{2.4\}$

$\{3.4\}.\ \{1.2.3\}.\ \{2.3.4\}.\ \{1.3.4\}.\ \{1.2.4\},\ \{1.2.3.4\}$

For, the numbers of a power set is $16 := 2^4$