Reminder III

- Special Gaussian probability distribution : $\mathbf{0} = N(0,0)$.
- Gaussian vector X if $a \cdot X$ is a Gaussian r.v.

  ($\equiv X_1, \ldots, X_N$ jointly Gaussian).
- X Gaussian vector $\Leftrightarrow \mathbb{E}(e^{aX}) = \exp(a \cdot \mathbb{E}(X) + a^T \mathrm{Cov}(X) a)$

  $\forall a \in \mathbb{R}^N$. $\rightsquigarrow$ importance of mean value + covariance matrix
- X non-degenerate if $\det(\mathrm{Cov}(X)) \neq 0$.

  $\Leftrightarrow a \cdot X = 0 \Rightarrow a = 0 \in \mathbb{R}^N$ linear independence
- Joint prob. distribution of non-degenerate Gaussian vector

  $$\Pi_X(x) = \frac{1}{(2\pi)^{N/2} |P|^{1/2}} \exp\left(-\frac{1}{2}(x-\bar{x})^T P^{-1}(x-\bar{x})\right)$$

  $P = \mathrm{Cov}(X)$ $\quad \uparrow \mathbb{E}(X)$
- $(X_t)_{t \in J}$ with $X_t : \Omega \to \Lambda$ family of random variables.
- Gaussian process if $(X_{t_1}, X_{t_2}, \ldots, X_{t_N})$ Gaussian vector

  some authors assume it non-degenerate, some not

  $\forall t_j \in J$, $t_j < t_{j+1}$. $\rightsquigarrow$ Examples defined by $\mathbb{E}$ + Cov
- Filtration $(\mathcal{F}_t)_{t \in J}$ family of σ-subalgebras of $\mathcal{F}$.
- Stochastic process $\mathbf{X} = (\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \in J}, (X_t)_{t \in J})$

  $X_t : \Omega \to \Lambda$ $\mathcal{F}_t$-measurable.