Reminder XIV

- Completeness of market model : $\exists \mathbb{P}^*$, all European option are attainable. Completeness $\Rightarrow$ $!\mathbb{P}^*$. (unique)

How to construct replicate strategy H?

- Generalized Black-Scholes model : for $i = 1, \dots, n$ (# risky assets)

$$dS_t^i = S_t^i b_i(t, S_t)\, dt + \sum_{j=1}^{N} S_t^i \sigma_{ij}(t, S_t)\, dB_t^j$$

(degrees of freedom) + regularity conditions.

- $N = n$ + $\sigma\sigma^T$ uniformly elliptic $\Rightarrow$ completeness + right price for all European options.

- If $r_t = r(t, S_t)$ (spot rate), $Z = h(S_T)$ (payoff), then

$$V_t(H) = \mathbb{E}^*\left(e^{-\int_t^T r(0, s_0)\, du} h(S_T) \,|\, \mathcal{F}_t \right) \overset{\text{Doob Dynkin lemma}}{=} P(t, S_t)$$

with P solution of a PDE, with $P(T, x) = h(x)$. (terminal value)

- $H_t^i = [\partial_{x_i} P](t, S_t) \quad \forall i = 1, \dots, n$. $H_t^0 = \dots$

other derivatives of P $\rightsquigarrow$ Greeks

- Black-Scholes model : $N = n = 1$, $\sigma > 0$.

$\rightsquigarrow$ everything computable explicitly ...