Reminder XI

- Markov property : $\mathbb{E}(f(X_t) \mid \mathcal{F}_s) = \mathbb{E}(f(X_t) \mid X_s)$

for any $t \geq s \geq 0$, $f \in M_b(\Lambda)$, $X_t : \Omega \to \Lambda$

↑ measurable + bounded functions ↑ standard measurable space

depends only on the difference t-s →

- Homogeneous Markov property : $\mathbb{E}(f(X_t) \mid \mathcal{F}_s) = \mathbb{E}(f(X_{t-s}) \mid X_0)$

- Markov transition function $p : \mathbb{R}_+ \times \mathbb{R}_+ \times \Lambda \times \mathcal{E} \to [0,1]$

↑ satisfies the Chapman-Kolmogorov relation.

Markov process associated to p: $\mathbb{E}(f(X_t) \mid X_s) = \int_\Lambda f(x)\, p(s,t,X_s,dx)$

$\rightsquigarrow p(s,t,y,A) = \mathbb{E}(\mathbb{1}_A(X_t) \mid X_s = y) = \mathbb{P}(X_t \in A \mid X_s = y)$.

If X homogeneous : $p(s,t,y,A) = p(0,t-s,y,A) =: p(t-s,y,A)$

- Time homogeneous diffusion process $\rightsquigarrow$ homogeneous Markov process (in-homogeneous $\rightsquigarrow$ not homogeneous M.p.)

- $\exists$ strong Markov property ...

- Feller process if p has some continuity

- For homogeneous Markov p. associated to p

$[U_t f](y) := \int_\Lambda f(z)\, p(t,y,dz)$, $f \in M_b(\Lambda)$

↑ contraction semi-group, leaving $C_0(\Lambda)$ invariant if Feller.