Homework 9

Exercise 1 *Consider $f:[a,b]\to\mathbb{R}$ continuous, and differentiable on (a,b), and suppose that f' is also continuous on $[a,b]$. Show that the length ℓ of the curve defined by $\big\{\big(x,f(x)\big)\mid x\in[a,b]\big\}$ is given by the expression*

$$\ell=\int_a^b\sqrt{1+f'(x)^2}\,\mathrm{d}x\ .$$

Exercise 2 *Let $f:[a,b]\to\mathbb{R}_+$ be continuous and consider the volume of revolution generated by the rotation of $\big\{\big(x,f(x)\big)\mid x\in[a,b]\big\}$ around the x-axis. Show that the volume V of this solid is given by the expression*

$$V=\pi\int_a^b f(x)^2\,\mathrm{d}x$$

Exercise 3 *Let $f:[a,b]\to\mathbb{R}_+$ be continuous, and differentiable on (a,b), and suppose that f' is also continuous on $[a,b]$. Consider the surface of revolution generated by the rotation of the points $\big\{\big(x,f(x)\big)\mid x\in[a,b]\big\}$ around the x-axis. Show that the surface S is given by the expression*

$$S=2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}\,\mathrm{d}x\ .$$

Exercise 4 (The painter's paradox) *Consider the function $f:[1,b]\ni x\mapsto\frac{1}{x}\in\mathbb{R}_+$ with $b>1$. In the setting of the previous two exercises show that for any $b>1$ one has $V=\pi\big(1-\frac{1}{b}\big)$ while $S>2\pi\ln(b)$. By considering the limit $b\to\infty$ why do we get an apparent paradox ?*