Nagoya University, G30 program Fall 2023

Mathematics Tutorial Ia Instructor : Serge Richard

Homework 3

Exercise 1 *Compute the following limits, if they exist:*

1. $\lim_{x\to 0_-}\left(\frac{1}{x}+\frac{1}{|x|}\right)$ *and* $\lim_{x\to 0_+}\left(\frac{1}{x}+\frac{1}{|x|}\right)$,
2. $\lim_{x\to 2_+}\frac{x^2+x-6}{|x-2|}$ *and* $\lim_{x\to 2_-}\frac{x^2+x-6}{|x-2|}$,
3. $\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}$.

Exercise 2 *Let I be an open interval in $\mathbb{R}$, and let $f: I\to\mathbb{R}$ be a continuous function. If $f(x)\neq 0$ for some $x\in I$, show that there exists $\delta>0$ such that $f(x+h)\neq 0$ for any $h\in[-\delta,\delta]$.*

Exercise 3 *From its definition, determine the slope of the tangent at each point of the graph of the function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=2x^2-3x+2$. Determine also the equation of the tangent at each point of the graph.*

Exercise 4 *Consider the function f defined by $f(x)=\sum_{n=0}^{\infty}\frac{1}{n!}x^n$ for any $x\in\mathbb{R}$.*

1. *For any fixed $x\in\mathbb{R}$ show that the sum is convergent,*
2. *Compute the derivative of f,*
3. *What can you say about this function ?*

Exercise 5 *We say that a function $f:\mathbb{R}\to\mathbb{R}$ is* uniformly continuous *if for any $x\in\mathbb{R}$ and any $\varepsilon>0$ there exists $\delta>0$ (which depends on f and ε but NOT on x) such that $|f(x+h)-f(x)|\leq\varepsilon$ for any $|h|\leq\delta$. Show that the function defined by $f(x)=x$ is uniformly continuous, but that the function defined by $f(x)=x^2$ is not uniformly continuous.*