Nagoya University, G30 program Fall 2023

Mathematics Tutorial Ia Instructor : Serge Richard

Homework 2

Exercise 1 *Consider the expression* $\frac{x^2+3x+2}{x^2+x-2}$. *Determine for which* $x \in \mathbb{R}$ *this expression is well-defined, and simplify this expression. Determine the maximal domain on which the function* $x \mapsto \frac{x^2+3x+2}{x^2+x-2}$ *can be defined.*

Exercise 2 *A function* $f : \mathbb{R} \to \mathbb{R}$ *is said to be* even *if* $f(-x) = f(x)$ *for any* $x \in \mathbb{R}$, *and* f *is said to be* odd *if* $f(-x) = -f(x)$ *for any* $x \in \mathbb{R}$.

1) Determine which of the functions defined for $x \in \mathbb{R}$ *by*

$$a)\ f(x) = x, \quad b)\ f(x) = x^2, \quad c)\ f(x) = x^2 + x, \quad d)\ f(x) = \sin(x), \quad e)\ f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

are even or odd ?

2) Show that for any function f, *the function* $g : \mathbb{R} \to \mathbb{R}$ *defined by* $g(x) = \frac{1}{2}\big(f(x) + f(-x)\big)$ *is an even function while the function* $h : \mathbb{R} \to \mathbb{R}$ *defined by* $h(x) = \frac{1}{2}\big(f(x) - f(-x)\big)$ *is an odd function. In addition, observe that* $f = g + h$.

3) What can you say about the graph of an even function, and about the graph of an odd function ?

Exercise 3 *Consider* $f : \mathbb{R} \ni x \mapsto x^2 \in \mathbb{R}$ *and* $g : \mathbb{R}_+ \ni x \mapsto \sqrt{x} - 1 \in \mathbb{R}$, *and determine if the following functions are well defined :*

$$a)\ f \circ g, \quad b)\ f \circ f, \quad c)\ g \circ g.$$

If one function is not well defined, on which maximal domain can one define it ?

Exercise 4 *Sketch the graph of the following function* $f : \mathbb{R} \to \mathbb{R}$ *given for* $x \in \mathbb{R}$ *by*

$$a)\ f(x) = \frac{1}{2}x - 2, \quad b)\ f(x) = x - 3, \quad c)\ f(x) = x^2 + 3x + 2, \quad d)\ f(x) = \begin{cases} 1 + 1/(x-2) & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}.$$

Exercise 5 *Determine the equation of the function* $f : \mathbb{R} \to \mathbb{R}$ *whose graph is a straight line containing the points* (x_1, y_1) *and* (x_2, y_2) *of* $\mathbb{R}^2$. *What is the slope of this line ?*

Exercise 6 *Let* $f, g : \mathbb{R} \to \mathbb{R}$ *be two continuous functions. Show as precisely as possible that*

1. *the sum* $\lambda f + g$ *is continuous on* $\mathbb{R}$ *for any* $\lambda \in \mathbb{R}$,
2. *the product* fg *is continuous on* $\mathbb{R}$.