Report

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1 Proof of Exercise 2.6.1

For an arbitrary normalized state $\big|f\rangle \in \mathscr{H}$, $\big|f\rangle\langle f\big|$ is a pure state. According to the definition of rays, we can use $\big|f\rangle$ to define a one-dimensional vector space $\hat{f} = \{\lambda\big|f\rangle \quad | \quad \lambda \in \mathbb{C}\}$. One can notice that this space is unique.

Conversely, if we first define the ray $\hat{f} = \{\lambda\big|f\rangle \quad | \quad \lambda \in \mathbb{C}\}$ with some normalized state $\big|f\rangle$, we can use the vectors in $\hat{f}$ that have unit length to construct pure states that has the form $\big|f\rangle\langle f\big|$. However, due to the calculation rule of coefficients of states in $\mathscr{H}$, the effect of phase does not matter, and we have constructed a unique pure state out of an element $\hat{f} \in \hat{\mathscr{H}}$.

Then we can find that the rays and pure states are in one-to-one correspondence, in other words, bijection.