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Exercise 1.3.3

We need to prove the following two things.

i) $SO(n) \cap \{\mathbb{1}, -\mathbb{1}\} = \{\mathbb{1}\}$

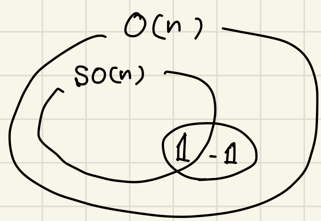
ii) For any M in $O(n)$, $\exists M' \in SO(n)$ s.t. $M = \pm\mathbb{1} \cdot M'$

i)

When n is odd,

$\det \mathbb{1} = 1$, $\det(-\mathbb{1}) = (-1)^n = -1$

Therefore, $SO(n) \cap \{\mathbb{1}, -\mathbb{1}\} = \{\mathbb{1}\}$



ii)

Using $\pm 1 = 1 \cdot (\pm 1)$ and the property of determinant,

($|AB| = |A||B|$ for any $A, B \in GL(n, \mathbb{R})$)

$\forall M \in O(n)$, $\exists M' \in SO(n)$ s.t. $M = \pm\mathbb{1} \cdot M'$

Therefore, for n odd, show that $O(n)$ is isomorphic to

$SO(n) \times \{\mathbb{1}, -\mathbb{1}\}$ $\square$

$O(n) \not\cong SO(n) \times \{\mathbb{1}, -\mathbb{1}\}$ if n is even, because $|-\mathbb{1}| = 1$

so $O(n) \cap SO(n) \times \{\mathbb{1}, -\mathbb{1}\} \neq \{\mathbb{1}\}$.