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Exercise 1.2.10

(Proof of proposition 1.2.9)

1. G_0 is a nomal subgroup of G $\Rightarrow$ $cG_0c^{-1} = G_0, \forall c \in G$

$\Rightarrow cG_0 = G_0c, \forall c \in G$

$\Rightarrow {}_{G_0}[c] = [c]_{G_0}, \forall c \in G.$

${}_{G_0}[a] = [a]_{G_0} \; \forall a \in G \Rightarrow aG_0 = G_0a, \forall a \in G$

$\Rightarrow aG_0a^{-1} = G_0, \forall a \in G.$

$\Rightarrow$ G_0 is a nomal subgroup of G

Therefore,

"G_0 is a nomal subgroup if and only if ${}_{G_0}[a] = [a]_{G_0}$ for any $a \in G$." $\square$

2. I only have to show that the composition law of G/G_0 is associativity.

$([a]_{G_0}[b]_{G_0})[c]_{G_0} = [ab]_{G_0}[c]_{G_0} = [(ab)c]_{G_0} = [abc]_{G_0}.$ ... ①

$[a]_{G_0}([b]_{G_0}[c]_{G_0}) = [a]_{G_0}[bc]_{G_0} = [a(bc)]_{G_0} = [abc]_{G_0}$ ... ②

($\because$ Because $a, b, c \in G$, $(ab)c = a(bc) = abc$)

①, ② $\Rightarrow$ The composition law of G/G_0 is associativity.

Therefore, G/G_0 is group. $\square$

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Assume there are R elements in G_0 and

m elements in G/G_0

Let $w_j\ (1 \le j \le R)$ be elements of G_0

and $v_i\ (1 \le i \le m)$ be boxes of G/G_0.

$$|\{v_1 w_1, v_1 w_2, \cdots, v_1 w_R\}| = R$$
$$|\{v_2 w_1, v_2 w_2, \cdots, v_2 w_R\}| = R$$
$$\vdots$$
$$|\{v_m w_1, v_m w_2, \cdots, v_m w_R\}| = R$$

and $G = \{v_1 w_1, v_1 w_2, \cdots, v_1 w_R\} \cup \cdots \cup \{v_m w_1, v_m w_2, \cdots, v_m w_R\}$.

(for any $(i_1, j_1) \ne (i_2, j_2)$, $v_{i_1} w_{j_1} \ne v_{i_2} w_{j_2}$)

Therefore,

$$|G| = mR = |G/G_0||G_0|$$

$$\Leftrightarrow |G/G_0| = \frac{|G|}{|G_0|} \qquad \square$$