Reminder VII

- Wigner's theorem : Symmetries are implemented by unitary or anti-unitary operators only. These operators are unique, up to multiplicative constant in $\mathbb{T} = \{z \in \mathbb{C} \mid |z| = 1\}$.

- Group of symmetries : $S(a)S(b) = S(ab)$, $S(e) = \mathbb{1}$, the corresponding (anti)-unitary operators satisfy

$$U(a)U(b) = \underbrace{\omega(a,b)}_{}\overset{\in \mathbb{T}}{} U(ab).$$

called 2-cocycle

This leads to:

- Projective representation : $(\mathcal{V}, U)$ with $U(a)U(b) = \omega(a,b)U(ab)$ for some $\omega : G \times G \to \mathbb{C}^*$. The following relation holds:

$$\omega(a,b)\,\omega(ab,c) = \omega(a,bc)\,\omega(b,c), \qquad \forall a,b,c \in G.$$

- All this related to group cohomology and to the notion of universal cover of G.

- Topological space $(\mathcal{M}, \mathcal{T})$, neighborhood of a point, Hausdorff property, basis, second countable.

• $(\mathcal{M}, \mathcal{T})$, $(\mathcal{N}, \mathcal{S})$ 2 topological spaces, and

$f : \mathcal{M} \to \mathcal{N}$ is <u>continuous</u> if $f^{-1}(U) \in \mathcal{T}$ $\forall U \in \mathcal{S}$

⚠ it does not mean f invertible

• If f is bijective and bi-continuous, then $(\mathcal{M}, \mathcal{T})$ and

$(\mathcal{N}, \mathcal{S})$ are <u>homeomorphic</u>, and f is a <u>homeomorphism</u>.

• <u>Topological manifold</u> (of dimension n and without boundary)

is a Hausdorff, second countable top. space $(\mathcal{M}, \mathcal{T})$

such that $\forall p \in \mathcal{M}$, $\exists$ a neighborhood V of p and

$\varphi : V \to \mathbb{R}^n$ injective and continuous, with $\varphi(V)$ open

and $\varphi^{-1} : \varphi(V) \to \mathcal{M}$ also continuous.