Reminder X

- Representation of a Lie algebra L : homomorphism $h : L \to \mathcal{L}(V)$
  preserves the structure
- A representation of a Lie group $\Rightarrow$ representation of its Lie algebra, the converse is only partially true.
- $U(a)\, e^{-itH} = e^{-itH}\, U(a) \quad \forall a \in G \Rightarrow$ cst of motion.
- Complexification of real Lie algebras L : 1) If linear independence over $\mathbb{C}$, just replace $\lambda \in \mathbb{R}$ by $\lambda \in \mathbb{C}$. 2) Otherwise, construction with pairs $(X, Y) \in L \times L$. $\rightsquigarrow$ complex Lie algebra $L_{\mathbb{C}}$ of the same dimension. Real form.
- Lie subalgebra, invariant Lie subalgebra $\equiv$ ideal.
- G_0 normal Lie subgroup of $G \Leftrightarrow L(G_0)$ ideal in $L(G)$.
  not an easy notion, ⚠
- Simple (semi-simple) Lie algebra : no invariant (Abelian) Lie subalgebra.
  non trivial and proper are assumed
- Simple (semi-simple) Lie group : Connected and non-Abelian Lie group with no normal, connected and closed (Abelian) subgroup.

Thm :

- A connected Lie group is (semi-) simple if and only if its Lie algebra is (semi-) simple.

- Adjoint representation of a Lie algebra L: $\forall X \in L$, set $ad_X : L \to L$ by $ad_X(Y) := [X, Y]$. Then

linear operators on L

$ad_X \in \mathcal{L}(L)$. Also: $ad_0 = 0$, $ad_{X+\beta X'} = ad_X + \beta ad_{X'}$ and $ad_{[X,Y]} = [ad_X, ad_Y]$. $\rightsquigarrow$ representation of L in $\mathcal{L}(L)$.