Reminder I

- <u>Group</u> : set G + law $G \times G \to G$ satisfying associativity, existence identity, existence inverse.

- <u>Abelian</u> ≡ <u>commutative</u> group, <u>finite</u> group.

- Examples : $(\mathbb{R}^n, +)$, C_n (cylic group), S_n (symmetric group), $GL(n,\mathbb{R})$, $SL(n,\mathbb{R})$
  $O(n)$, $SO(n)$, $GL(n,\mathbb{C})$, $SL(n,\mathbb{C})$, $U(n)$, $SU(n)$.
  $U^* = U^{-1} \Rightarrow |\det(U)| = 1$

- <u>Subgroup</u>.

- <u>Conjugation</u> : $b = cac^{-1}$, $b \sim a$ (equivalence relation), $[a]$ conjugacy class

- Subgroup $G_0 \subset G$ <u>normal</u> if $cG_0c^{-1} = G_0$ $\forall c \in G$. $\hookrightarrow G_0 \triangleleft G$

- <u>Simple</u>, <u>semi-simple</u> group
  Simple: no proper and non-trivial normal subgroup
  Semi-simple: no proper and non-trivial normal Abelian subgroup

- <u>Left</u>, <u>right</u> <u>conjugation</u> : $b \in aG_0$, $b \overset{\ell}{\sim} a$, ${}_{G_0}[a]$
  $b \in G_0 a$, $b \overset{r}{\sim} a$, $[a]_{G_0}$

- <u>Thm</u> : ${}_{G_0}[a] = [a]_{G_0}$ iff G_0 normal. If so G/G_0 is a group, with $[a]_{G_0}[b]_{G_0} = [ab]_{G_0}$.
  ↑ quotient group, or factor group