Nagoya University, G30 program Fall 2022

Mathematics Tutorial Ia Instructor : Serge Richard

Homework 8

Exercise 1 *Compute*

a) $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln(x)}\right)$,

b) $\lim_{x\to 0_+}\big(1+\sin(4x)\big)^{\cot(x)}$ *with* $\cot(x)=\frac{1}{\tan(x)}$.

Exercise 2 *Consider the function* $\tanh:\mathbb{R}\to(-1,1)$. *Show that this function is invertible and compute the derivative of its inverse.*

Exercise 3 *Show that* $\tanh(y)^{-1}=\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$ *for any* $y\in(-1,1)$.

Exercise 4 *Differentiate the function* $\mathbb{R}_+\ni x\mapsto\frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\in\mathbb{R}_+$.

Exercise 5 (Midterm 2021) *1) Consider* $f:[a,b]\to[\alpha,\beta]$ *a strictly increasing and continuous function satisfying* $f(a)=\alpha$ *and* $f(b)=\beta$. *Show that its inverse function* $f^{-1}:[\alpha,\beta]\to[a,b]$ *is continuous at any* $y\in(\alpha,\beta)$. *Hint: Consider* $y\in(\alpha,\beta)$, *set* $x:=f^{-1}(y)$, *and let* $\epsilon>0$ *such that* $[x-\epsilon,x+\epsilon]\subset(a,b)$. *Set* $\delta:=\min\{|y-f(x-\epsilon)|,|y-f(x+\epsilon)|\}$, *and... show the statement.*

2) Observe that the continuity of f *is not necessary in the previous proof. In fact, the following statement holds: If* $f:[a,b]\to\mathbb{R}$ *is strictly increasing (but not necessarily continuous), its inverse* $f^{-1}:f\big([a,b]\big)\to\mathbb{R}$ *is continuous. Can you provide an example of this situation, and explain it ?*