
Homework 4

Exercise 1 Compute the derivative of the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ with $f(x)$ provided by the following expressions:

$$a) 5x^4 + 4x^2 - 1, \quad b) (x^5 + 1)(x^2 - 1), \quad c) \frac{5x - 1}{x - 5} \text{ for } x \neq 5, \quad d) \frac{x^{25} - 2x}{x^2 + 3}.$$

Exercise 2 By using that $\sin'(x) = \cos(x)$ show that $\cos'(x) = -\sin(x)$ for any $x \in \mathbb{R}$.

Exercise 3 Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 \sin(1/x)$ if $x \neq 0$ and $f(0) = 0$.

1. Show that f is continuous at 0,
2. Compute the derivative of f at 0,
3. Compute the derivative of f at any $x \neq 0$,
4. Show that the derivative of f is well-defined but that this derivative is not continuous at 0.

Indication: you can use that $\lim_{y \rightarrow 0} \frac{\sin(y)}{y} = 1$.

Exercise 4 By using the indication mentioned above, compute the following limits:

1. $\lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h}$,
2. $\lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h^2}$.

Exercise 5 a) Let $f(x) = x^2 \sin(1/x)$ and $g(x) = \sin(x)$ for any $x \in (-1, 0) \cup (0, 1)$. Show that $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$ does not exist, but that $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = 0$.

b) Explain how this example fits in with L'Hospital's rule ?

Exercise 6 Compute the derivatives of order 1, 2 and 3 for the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ defined for $x \in \mathbb{R}$ by

$$a) \cos(x) \quad b) \cos(x) \sin(x) \quad c) x^4 + x^3 + x^2 + x^1 + 1$$