Assessment Test in Mathematics for the G30 program

29th September 2021, 13:00–14:00.

- The test should be completed with manual writing material or on an tablet – calculators, mobile phones, etc. are not allowed.
- For multiple choice questions, write down *only* the letter indicating your answer.
- For other questions, give solutions and/or justifications, together with a clear *answer* to the question asked.
- After finishing scan and send your solution as one pdf to henrik.bachmann@math.nagoya-u.ac.jp

Problem 1. The fraction $\dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{2}}$ is equal to

a) $\sqrt{2}$ **b)** $\sqrt{3}-1$ **c)** $\dfrac{\sqrt{3}+2}{2}$ **d)** $\sqrt{3}+1$ **e)** $\dfrac{\sqrt{6}+2}{2}$

Problem 2. A man can do a job in n hours alone, and his son can do it in $2n$ hours alone. Together, how many hours will it take them to do the job?

a) $\dfrac{1}{3}n$ **b)** $\dfrac{3}{2}n$ **c)** $\dfrac{1}{2}n$ **d)** $\dfrac{2}{3}n$ **e)** $3n$

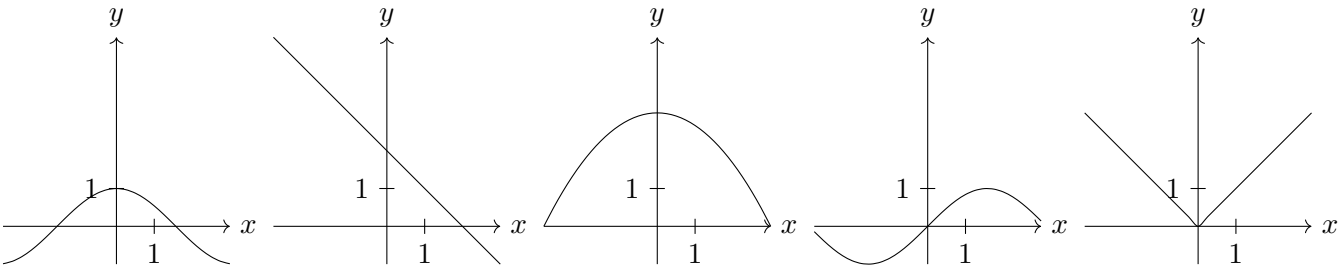
Problem 3. If x, a, b are positive numbers, then $\left(x^a\right)^b$ is equal to

a) x^{a^b} **b)** $x^{a/b}$ **c)** x^{ab} **d)** x^{a+b}

Problem 4. Each of the functional expressions (a)–(e) corresponds to one of the graphs below. Indicate which by pairing each letter with the number of the corresponding graph.

a) $y = 3 - \dfrac{x^2}{3}$ **b)** $y = |x|$ **c)** $y = 2 - x$

d) $y = \cos(x)$ **e)** $y = \sin(x)$



(1) **(2)** **(3)** **(4)** **(5)**

Problem 5.

- Class A has a higher enrollment than Class B.
- Class C has a lower enrollment than Class B.
- Class A has a lower enrollment than Class C.

If the first two statements are true, the third statement is

a) false **b)** true **c)** uncertain

Problem 6. We consider an island called *Sugashima*. Given the following two statements:

"*at Sugashima, all mangoes are golden in colour*", and
"*at Sugashima, no golden-coloured things are cheap*",

which of the following statements are true, false, respectively uncertain?

*Remark: The above statements do not mean that there are any mangoes at Sugashima. Therefore, for example, the statement "*at Sugashima, mangoes exist*" would be uncertain.*

Write **T** for "true", **F** for "false", and **U** for "uncertain".

1. At Sugashima, there are no golden-coloured pears.
2. At Sugashima, a cheap car cannot be golden-coloured.
3. At Sugashima, all golden-coloured fruits are mangoes.
4. At Sugashima, golden-coloured mangoes are not cheap.
5. At Sugashima, all mangoes are cheap.
6. At Sugashima, there exists a cheap mango.
7. At Sugashima, all green fruits are cheap.

Problem 7. What is the slope of the line containing the points $(-4, 1)$ and $(2, 3)$?

Problem 8. Which of the following equations describe the straight line in the x/y-plane that contains the points $(15, -1)$ and $(9, 2)$?

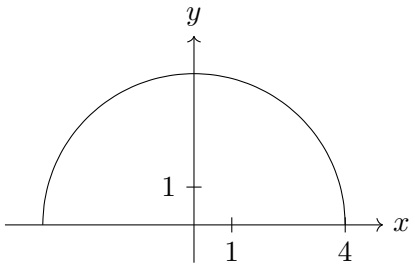
a) $x - 2y = 17$ **b)** $x + 3y = 12$ **c)** $2x - y = 16$ **d)** $x + 2y = 13$

Problem 9. Find $f(-3)$ for $f(x) = x^3 - 1$.

Problem 10. Find $g(2a)$ for $g(x) = x^2 - 1$.

a) $2a^2 - 2$ **b)** $2a^2 - 1$ **c)** $4a^2 - 1$ **d)** $4a^2 - 2$

Problem 11. On which interval is the function, whose graph is given below, increasing?



a) $(-4, 4)$ **b)** $(-4, 0)$ **c)** $(0, 4)$ **d)** $(-5, 5)$

Problem 12. The graph of $y = f(x + 1) - 3$ is the graph of $y = f(x)$ ...

a) ... shifted to the right 1 unit and up 3 units.

b) ... shifted to the left 3 units and down 1 unit.

c) ... shifted to the left 1 unit and down 3 units.

d) ... shifted to the right 3 units and up 1 unit.

Problem 13. Let $f(x) = -3(x - 4)^2 + 7$. Which of the statements below is true?

a) The minimum value of f is 7. **b)** The maximum value of f is 10.
c) The maximum value of f is 7. **d)** The minimum value of f is 5.

Problem 14. Solve the equation $x^2 + x - 6 = 0$.

Problem 15. For any real numbers a and b, $(a + 3b)^2$ is equal to:

a) $a^2 + 9b^2$ **b)** $a^2 + 4ab + 3b^2$ **c)** $a^2 + 3b^2$ **d)** $a^2 + 6ab + 9b^2$

Problem 16. Compute the following limits.

1. $\lim\limits_{x \to +\infty} \dfrac{5x - 3}{x + 4}$
2. $\lim\limits_{x \to 1^-} \dfrac{1}{1 - x}$

Problem 17. The value of $e^{3\ln(2) - \ln(5)}$ is equal to

a) $\dfrac{8}{5}$ **b)** 1 **c)** $\dfrac{5}{8}$ **d)** 3

Note: ln *is the function called the "natural logarithm".*

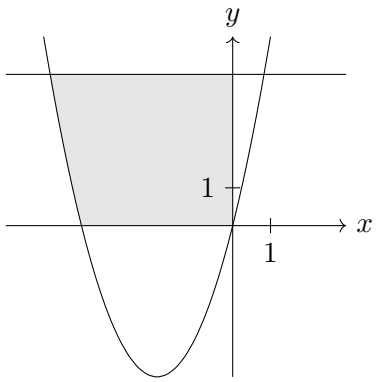
Problem 18. Solve the following system of equations:

$$\begin{cases} 2x + y = 1 \\ x + 4y = 6 \end{cases}$$

Problem 19. Compute the following product of matrices:

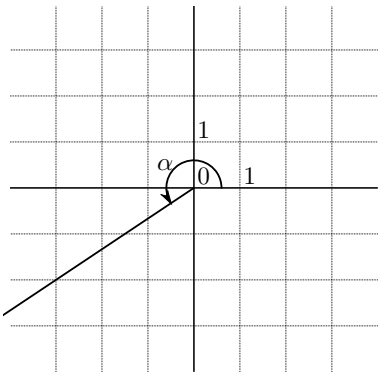
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & -2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

Problem 20. The grey area in the following figure marks the solutions to one of the systems of inequalities below. Which one?



a) $\begin{cases} y \leq x^2 + 4x \\ x \leq 0 \\ 0 \leq y \end{cases}$ **b)** $\begin{cases} y \geq x^2 + 4x \\ x \leq 0 \\ 0 \leq y \leq 4 \end{cases}$ **c)** $\begin{cases} y \leq x^2 + 4x \\ x \geq 0 \\ 0 \leq y \leq 4 \end{cases}$ **d)** $\begin{cases} y \geq x^2 + 4x \\ x \leq 0 \\ y \leq 4 \end{cases}$

Problem 21. Let α be the angle indicated in the following figure:



Give the values of:

1. $\sin(\alpha)$
2. $\cos(\alpha)$
3. $\tan(\alpha)$

Problem 22. Compute the derivatives of the following functions:

1. $f(x) = \sin(x)$
2. $g(x) = x^2 - 3x + 2$
3. $h(x) = \exp(2x^3)$
4. $k(x) = \ln(1 + x)$

Problem 23. Compute the following integrals.

1. $\int 3x^3 \, \mathrm{d}x$
2. $\int_0^{\pi/2} \sin(x) \, \mathrm{d}x$

Problem 24. Compute the following product of complex numbers: $(3 + 2i)(1 - 2i)$