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Report by Li Yucheng No.
Date.
$\frac{1}{2} \frac{1}{2} \frac{1}$
1 ACC/ACC/= A(S)/A(E), (X/O/= X.
We need to prove: X(t)= elotalsids x, is a solution -> A(t)A(s)=A(s)A(t)
TI I'M I'M ( tals) ds
The differential of x(t) = elotals) ds :  x(t) = lim elotals) ds = elotals) ds is:
= lim = = til [(/otherals)ds)k-(/otals)ds)k](1)
$if \times (t) = A(t) \times (t), then$
$if x(t) = A(t)x(t), then  x(t) = A(t) e^{-tA(s)ds}$
=A(t) = j! (/tA(s) ds) suppose j+1=k
= A(t) = (k-1)! (/o A(s) ds) k-1
$= \sum_{k \in I} \frac{1}{k!} k A(t) (( {}_{\circ}^{t} A(s) ds)^{k-1} - \cdots (z)$
We can see that it is necessary to make (1)=(2)
(/otA(s) ds) = (otA(s) ds) - (/otA(s) ds)
= fot A(s1) ds1 fot A(s2) ds2 - fot A(sk) dsk
= (10 the Als) ds - ( Als) ds) = = the si-sk are independent variables
$=\frac{1}{\varepsilon}\cdot\varepsilonA(t+0\varepsilon)=A(t+0\varepsilon)$
Then the till ( Als) ds) k - ( Als) ds) k]
= = [ ( Alsı)dsı - fot Alsk)dsk - fot Alsı)dsı - fot Alsk)dsk)
== 11t+Epic) de 1t+Epic) de - /t/e) de 1t+Epic) la 1t+Epic) la +/to) /
tte Also de - Also de - Also de Attente de Also de la transportario del transportario del transportario de la transportario del la transportario de la transportario del la transportario de la transportario de la transportario de la transportario de la transportario
= \frac{1}{(\sin Alsi) ds_1 \cdot \frac{t+e}{Alsi) ds_k - \frac{t}{alsi) ds_1} \frac{t+e}{Alsi) ds_2 \cdot \frac{t+e}{Alsi) ds_k - \frac{t}{alsi) ds_1} \frac{t+e}{Alsi) ds_2 \cdot \frac{t+e}{Alsi) ds_k - \frac{t}{alsi) ds_1} \frac{t+e}{Alsi) ds_2 \cdot \frac{t+e}{Alsi) ds_k \cdot \frac{t+e}{Alsi) ds_2 \cdot \frac{t+e}{Alsi) ds_k \cdot \frac{t+e}{Alsi} \frac{t+e}{A
[Alsk-1)dsk-1/6 Alsk)dsk-/oAlsi)dsi-/tAlsk)dsk)(3)

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Denote / tte A(sj) dsj - / tA(sj) dsj
            = \int_{a}^{t+\epsilon} A(sj) dsj = R(t, \epsilon)
 Due to the definition of integral,
we can consider Alsj) to be constant on [1.t+ \varepsilon] when \varepsilon > 0, which is obviously
A(t), So
limer(t,c) = lim = E.EA(t) = A(t)
(', (3) = \frac{1}{\epsilon} [R(t, \epsilon)]_{s}^{t+\epsilon} A(s_1) ds_2 - \int_{s}^{t+\epsilon} A(s_k) ds_k + \cdots
                   + /t A(s1) ds1--- /t A(sx-1) dsx-1 R(t, E)]
lim (4) = lim = [ & Alt)/ = Alsz) dsz.../ te Alsk) dsk+...

E->0
+E/ t Alsz) dsz. - / t Alsk-1) dsk-1 Alt)]
          - A(t)/ A(sz) dsz. -/ A(sx) dsz + · · ·
              + /stA(s1)ds1---/,tA(sn-1)dsn-1 Alt)
          = A(t) (/otA(s)ds)k-1 + /otA(s)ds A(t) (/otA(s)ds)k-2
              + - - + (/2+Als) ds) +- A(t)
                                                   . _^ -. (5)
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Take (5) into (1), we can have  $\dot{x}(t) = \sum_{k=0}^{\infty} t_k! (5)$  (5) We can easily see that this equals to (2) only if A(s) and A(t) commute so that we can move A(t) to the left side of every term.

Thus,  $\dot{x}(t) = A(t)e^{-t}A(s)ds = A(t)x(t)$  only if A(t)A(s) = A(s)A(t).