Derivative of the Inverse Function

FIRDAUS Rafi Rizqy / 062101889

Calculus 1 (Fall 2021)

1 Problem

The derivative of an inverse of a continuous and an invertible function $f : [a, b] \to \mathbb{R}$ and differentiable on (a, b). Set $\alpha :=$ f(a) and $\beta :=$ f(b), and let $f^{-1} : [\alpha, \beta] \to [a, b]$ be the inverse function. Then f^{-1} is continuous on $[\alpha, \beta]$ and differentiable on (α, β) with

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}, \ \forall y \in (\alpha, \beta)$$

2 Proof

Assume that f^{-1} is differentiable on (α, β). Thus, by using chain rule from **Proposition 2.17** $((g \circ f)'(x) = g'(f(x))f'(x))$ and the property of an inverse function $((f \circ f^{-1})(y) = y)$ one has

$$\begin{aligned} (f \circ f^{-}1)'(y) &= \frac{d}{dy}(y) \\ f'(f^{-1}(y))(f^{-1}(y))' &= 1 \\ (f^{-1}(y))' &= \frac{1}{f'(f^{-1}(y))} \end{aligned}$$

Thus, the inverse of function f is differentiable on (α, β) and the derivative is given by the equality as following

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}, \ \forall y \in (\alpha, \beta)$$

□