
Homework 9

Exercise 1 Find the area under the graph of the function mentioned below and between the given bounds:

1. $x \mapsto x^3$ between $x = 0$ and $x = 2$,
2. $x \mapsto e^{-x}$ between $x = 0$ and $x = b > 0$, what happens when $b \rightarrow \infty$?
3. $x \mapsto \cos(x) + \cos(2x)$ between $x = 0$ and $x = \pi/4$,
4. $x \mapsto x - \sin(x)$ between $x = 0$ and $x = \pi/2$,

and represent each of these areas on a drawing.

Exercise 2 Write out the lower and the upper Riemann sums for the function $x \mapsto x^2$ in the interval $[0, 2]$. Use a regular partition of the interval divided into n subintervals of the same length. The following formula can be used:

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

What happens when $n \rightarrow \infty$?

Exercise 3 Consider the function $[0, 1] \ni x \mapsto e^x \in \mathbb{R}$, and consider a regular partition of $[0, 1]$ divided into n intervals of length $\frac{1}{n}$. Compute the following Riemann sums:

1. $I_l := \sum_{j=0}^{n-1} \frac{1}{n} e^{\frac{j}{n}}$ left rule,
2. $I_r := \sum_{j=1}^n \frac{1}{n} e^{\frac{j}{n}}$ right rule,
3. $I_m := \sum_{j=0}^{n-1} \frac{1}{n} e^{\frac{j+1/2}{n}}$ midpoint rule,
4. $I_{tri} := \frac{1}{2}(I_l + I_r)$ trapezoidal rule.

Illustrate these rules on a drawing, and compute the limit of these expressions when $n \rightarrow \infty$. The following formula can be used for any $a > 0$ with $a \neq 1$:

$$\sum_{k=0}^{m-1} a^k = \frac{1 - a^m}{1 - a}.$$

Exercise 4 Write the Riemann sums for the function $x \mapsto (x^3 - 6x)$ on the interval $[0, 3]$, and consider the limit when the number of subintervals goes to infinity. You can use the two equalities:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2.$$