

**Homework 12**

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**Exercise 1** Provide the Taylor's expansion (for  $x$  around 0) of order  $n$  for the following functions, and provide an estimate on the remainder term:

1)  $(-1, \infty) \ni x \mapsto \ln(1+x) \in \mathbb{R}$ ,

2)  $(-1, 1) \ni x \mapsto (1+x)^s \in \mathbb{R}$  for any  $s \in \mathbb{R}$ ,

3)  $\mathbb{R} \ni x \mapsto \sin(x) \in \mathbb{R}$ ,

4)  $\mathbb{R} \ni x \mapsto \sinh(x) \in \mathbb{R}$ ,

5)  $\mathbb{R} \ni x \mapsto \cosh(x) \in \mathbb{R}$ ,

**Exercise 2** Provide the Taylor's expansion (for  $x$  around  $\pi/4$ ) of order  $n$  for the functions mentioned below, and provide an estimate on the remainder term:

$$\mathbb{R} \ni x \mapsto \cos(x) \in \mathbb{R}.$$

**Exercise 3** By using Taylor's expansions compute the following limits:

1.  $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x \tan(x)}$ ,

2.  $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{\sin(x)}$ ,

3.  $\lim_{x \rightarrow 0} \frac{\sin(x) - e^x + 1}{x}$ .